

ANALYSIS OF SINGLE PHASE SEMI-CONVERTER CONTROLLED DC MOTOR

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ABSTRACT

In modern DC motor drives, controlled rectifiers are popularly employed to control the speed of DC motors. Controlled rectifier-fed dc motor drives are applied where frequent starting, braking, and reversing are needed, for example, in rolling mills, paper mills, printing presses, mine winders, and machine tools. They produce a high starting torque, and provide speed control over a wide range of speed. The speed of a dc motor can be controlled by (i) controlling the armature voltage (V_a) - voltage control or (ii) controlling the field flux (ϕ) or field current (I_f) - field control and (iii) torque demand-corresponding to an armature current, I_a for a fixed field current, I_f . In this paper, single-phase, full-wave, half-controlled rectifier was used to control the speed of a separately-excited dc motor. This rectifier produces variable dc output voltage from a fixed alternating current (ac) voltage. The output of this rectifier supplied the armature of a separately excited dc motor. Another single-phase, full-wave, half-controlled rectifier supplied the field winding of the motor. The armature voltage (V_a) and field circuit voltage (V_f) were varied by varying the delay or firing angles, α_a and α_f respectively, of the rectifiers in order to increase or decrease the speed or torque of the motor. The results obtained show that (i) the speed of the motor (ω_m) varies directly with the armature voltage (V_a), (ii) the speed (ω_m) is inversely proportional to the field current (I_f), (iii) the torque (T_a) developed by the motor is directly proportional to the armature current (I_a) and the field current (I_f). At rated armature voltage (200 V), the no-load speed of the motor is 157 rad/s. At full load (9.55 N.m), the full-load speed is 110 rad/s.

I. INTRODUCTION

Electric machines play an important role in industry as well as our day to day life. They are used to generate electrical power in power plants and provide mechanical work in industries. They are also indispensable part of our daily lives. Industries use a dozen or more electric motors. D.C machines are one of the most commonly used machines for electromechanical energy conversion. Electrical machines convert electrical input to mechanical output or vice versa. If the conversion is from

mechanical to electrical, the machine is said to act as a generator. If the conversion is from electrical to mechanical, the machine is said to act as a motor (Daniel, 2011).

Converters are integral part of any power supply unit used in the all-electronic equipment with various applications in the industrial, domestic, agricultural. Application of dc motors include manufacture of pulp, paper and paper board, propulsion of electric vehicles, textile industries and public transportation, such as subway and trolley systems, machine tools, printing presses, electric traction, conveyors, fans, pumps, hoist and cranes due to the ability to yield itself to speed variation.. It can be used as an interface between utility and most of the power electronic equipment. These electronic equipment form a major part of load on the utility. Electric motors exist to convert electrical energy into mechanical energy. This is done by two interacting magnetic fields: one stationary and another attached to a part that can move (Sen, 1995). D.C. motors have the potential for very high torque capabilities (although this is generally a function of the physical size of the motor), (Donnelly, 1972), they are also easy to miniaturize, and can be “regulated” via adjusting their supply voltage. D.C. motors are not only simple, but the oldest electric motor (Khaled, 2013). Modern DC motor drives utilize power electronic devices and are subdivided into chopper-fed and controlled rectifier-fed drives. Small D.C machines (in fractional horse power rating) are used primarily as control devices for speed sensing and servomotors for positioning and tracking (Morgan, 1997).

The motor speed can be varied by (1) controlling the armature voltage, V_a known as *voltage control*; (2) controlling the field current, I_f known as *field control*; or (3) torque demand, which corresponds to an armature current, I_a , for a fixed field current, I_f . The speed, which corresponds to the rated armature voltage, rated field current and rated armature current is known as the *base speed*. In practice, for a speed less than the base speed, the armature current and field currents are maintained constant to meet the torque demand and the armature voltage (V_a) is varied to control the speed. For speed higher than the base speed, the armature voltage is maintained at the rated value and the field current is varied to control the speed.

II. MATERIALS AND METHODS

2.1 Operation of d. c. motors

Figure 1 is the steady-state equivalent circuit of the separately excited dc motor, (Dubey, 1989). The field and armature voltages are controlled independent of each other. Thus, when electric current I_f flows in the field winding, the magnetic field created exerts a force on the rotor or armature coils carrying the current (I_a), causing it to rotate. A voltage, E known as back EMF is induced in the armature winding as it rotates. V_a and R_a are the terminal voltage and resistance respectively of the armature circuit.

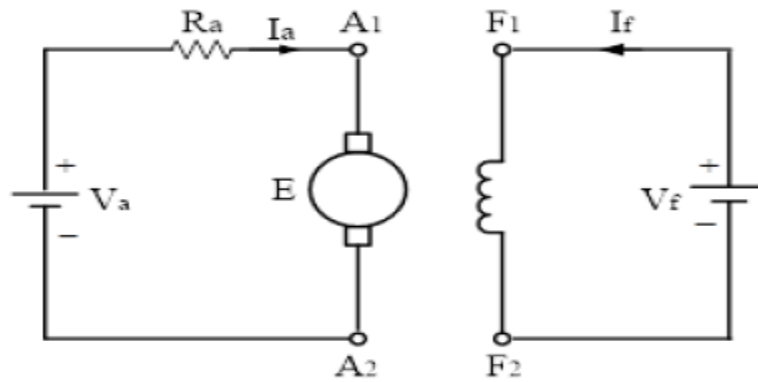


Fig. 1: Separately Excited DC Motor

From Figure 1, the basic equations of the DC motor for steady-state operation can be derived as follows, (Onah, et al, 2021):

$$V_a = R_a I_a + E \quad (1)$$

It can be shown that the relationship between the generated EMF, speed, flux, and the number of conductors in the armature is:

$$E = \frac{pZ}{c} \frac{2\pi N}{60} \phi = k \phi \omega_m = K \omega_m \quad (2)$$

The flux (ϕ) is proportional to the field current (I_f):

$$E = k I_f \omega_m = K \omega_m \quad (3)$$

Where

p = number of poles

N = speed in rpm

ϕ = useful flux per pole entering or leaving the armature in Wb

$$\omega_m = \frac{2\pi N}{60} \text{ speed in rad/s} \quad (4)$$

$$k = \frac{pZ}{c} = \text{Constant of every machine}$$

Z = total number of armature conductors

c = number of parallel paths through winding between positive and negative brushes

$\frac{Z}{c}$ = Number of conductors in series in each path

From equation (1) the total electrical power supplied to the armature is given by:

$$V_a I_a = R_a I_a^2 + EI_a \tag{5}$$

$R_a I_a^2$ = Loss due to resistance of armature circuit

$E I_a$ = Mechanical power developed by the armature

From equation (3.2):

$$P_m = EI_a = k I_f \omega_m I_a = T_a \omega_m \tag{6}$$

T_a = torque developed by the armature

Therefore

$$T_a = k I_f I_a = K I_a \tag{7}$$

Equation (1) can be written as:

$$V_a = R_a I_a + K \omega_m \tag{8}$$

So,

$$\omega_m = \frac{V_a - R_a I_a}{K} \tag{9}$$

From equations (7) and (9), the relationship between the motor speed and developed torque can be written as:

$$\omega_m = \frac{V_a}{K} - \frac{R_a}{K^2} T_a \tag{10}$$

2.2 Open loop transfer function

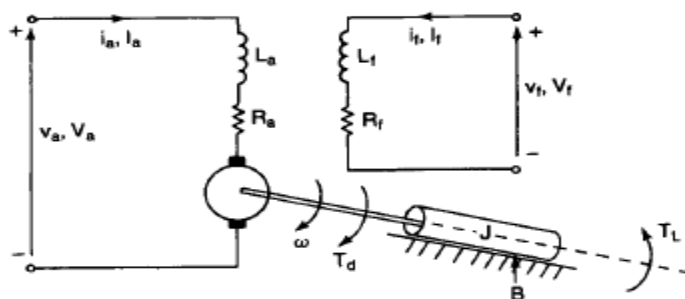


Fig. 2: Equivalent Circuit of Separately Excited DC Motors

The equivalent circuit for a separately excited dc motor is shown in Figure 2, (Rashid, 1993), (Malatestas, et al, 2013). When a separately excited motor is excited by a field current, I_f and armature current, I_a flows in the armature circuit, the motor develops a back emf, E and a torque to balance the load torque at a particular speed. The circuit arrangement of a separately excited dc motor drive with open-loop control is shown in Figure 2. If the field current I_f and back emf constant k are constant during any transient disturbances, the equations of the system in Figure 2 are:

$$V_f = R_f i_f + L_f \frac{di_f}{dt} \quad (11)$$

$$V_a = R_a i_a + L_a \frac{di_a}{dt} + e_a \quad (12)$$

$$T_m = k i_f i_a \quad (13)$$

$$e_a = k i_f \omega \quad (14)$$

e_a = back emf

The developed torque T_m is given as

$$T_m = J \frac{d\omega}{dt} + B\omega + T_L \quad (15)$$

J = inertia of rotor plus load

B = Viscous friction constant

T_L = Load torque

ω = motor speed, rad/s

In Laplace transform, the forgoing equations become:

$$V_a(s) = R_a I_a(s) + s L_a I_a(s) + k I_f \omega(s) \quad (16)$$

$$T_m(s) = k I_f I_a(s) \quad (17)$$

$$E_a(s) = k I_f \omega(s) \quad (18)$$

$$T_m(s) = s J \omega(s) + B \omega(s) + T_L(s) \quad (19)$$

From equation (15)

$$I_a(s) = \frac{V_a(s) - k I_f \omega(s)}{R_a + s L_a} \quad (20)$$

From equation (18)

$$\omega(s) = \frac{T_m(s) - T_L(s)}{sJ + B} \quad (21)$$

From equations (16), (17), (19) and (20), the open-loop block diagram of the separately excited dc motor drive is shown in Figure 3.

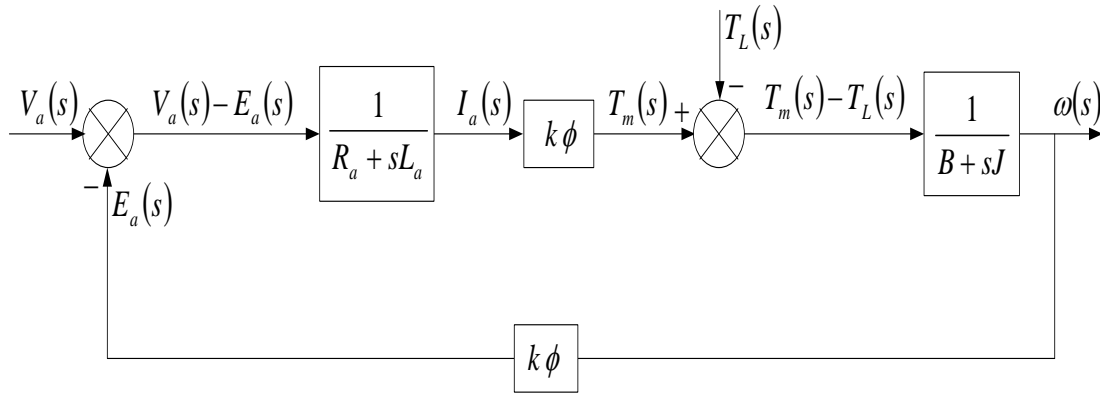


Fig. 3: Open-loop Block Diagram of Separately Excited DC Motor Drive

There are two possible disturbances (or inputs), $V_a(s)$ and $T_L(s)$. The steady-state responses can be determined by combining the individual response due to V_a and T_L .

The system response due to step change in V_a is obtained by setting $T_L = 0$. Therefore the relation between the input and output is:

$$[V_a(s) - k\phi\omega(s)] \left(\frac{1}{R_a + sL_a} \right) k\phi \left(\frac{1}{B + sJ} \right) = \omega(s) \quad (22)$$

$$\frac{\omega(s)}{V_a(s)} = \frac{k\phi}{JL_a s^2 + (JR_a + BL_a)s + BR_a + (k\phi)^2} \quad (23)$$

The response due to a change in load torque, T_L can be obtained by setting $V_a = 0$. The block diagram for this case is shown in Figure 4

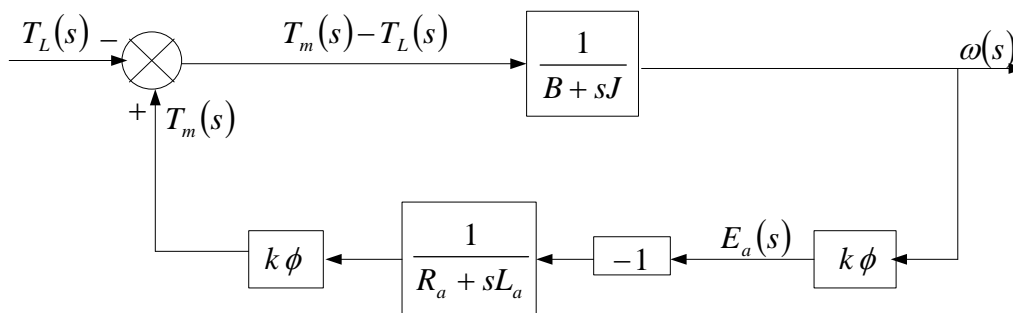


Fig. 4: Open-loop Block Diagram of Torque Disturbance Input

The relation between input and out is:

$$\left[-k\phi\omega\left(\frac{1}{R_a + sL_a}\right)k\phi - T_L \right] \left(\frac{1}{B + sJ}\right) = \omega$$

$$\frac{\omega(s)}{T_L(s)} = -\frac{R_a + sL_a}{JL_a s^2 + (JR_a + BL_a)s + BR_a + (k\phi)^2} \tag{24}$$

2.3 Single-phase semi-converter

Controlled rectifiers provide a variable dc output voltage from a fixed ac voltage. Due to their ability to supply a continuously variable dc voltage, controlled rectifiers made a revolution in modern industrial control equipment and variable-speed drives with power levels ranging from fractional horsepower to several megawatts. The circuit arrangement of a single-phase semi-converter, with a dc motor, is shown in Figure 5. (Rashid, 1993), (Kazuaki, et al, 2014), (Jha, et al, 2006), (Jomy, et al, 2009).

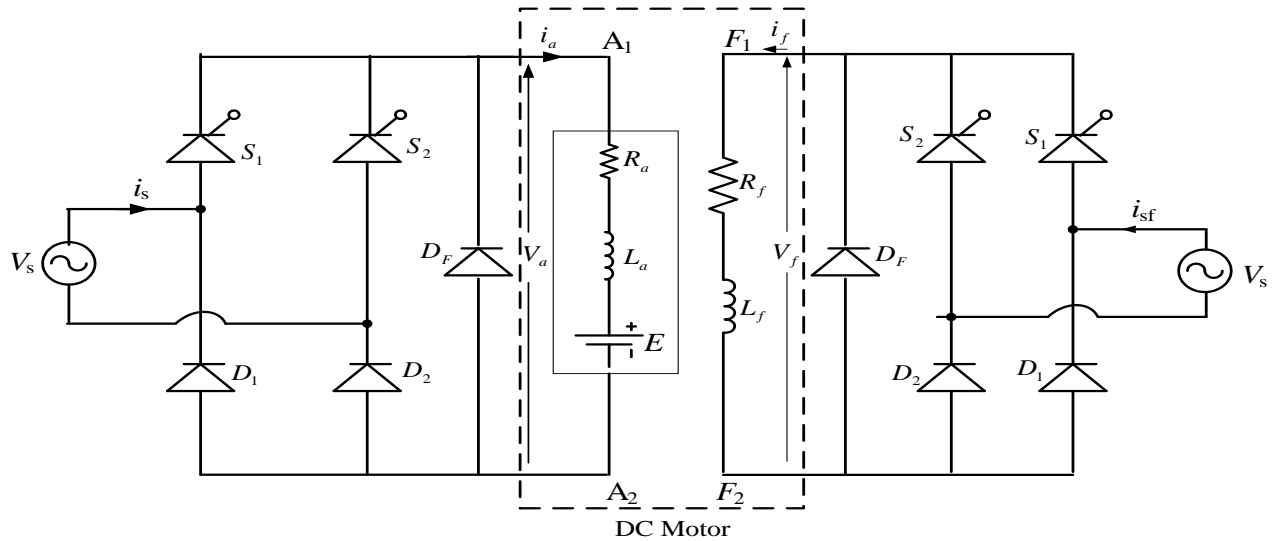


Fig. 5: Circuit of Single-phase Semi-Converter-Controlled DC Motor

During the positive half-cycle, thyristor T_1 is forward biased. When thyristor T_1 is fired at $\omega t = \alpha$, the load is connected to the input supply through T_1 and D_2 during the period $\alpha \leq \omega t \leq \pi$. During the period $\pi \leq \omega t \leq (\pi + \alpha)$, the input voltage is negative and the freewheeling diode D_m is forward biased. D_m provides continuity of current in the inductive load. The load current is transferred from T_1 and D_2 to D_m ; and thyristor T_1 and diode D_2 are turned off. During the negative half-cycle of input voltage, thyristor T_2 is forward biased, and firing of thyristor T_2 at $\omega t = \pi + \alpha$ will reverse bias D_m . The diode D_m is turned off and the load is connected to the supply through T_2 and D_1 .

$$V_o = \frac{1}{2\pi} \int_0^{2\pi} V_m \sin \omega t d\omega t = \frac{V_m}{2\pi} \left\{ \int_{\alpha}^{\pi} \sin \omega t d\omega t - \int_{\pi+\alpha}^{2\pi} \sin \omega t d\omega t \right\} = \frac{V_m}{\pi} (1 + \cos \alpha) \quad (25)$$

The average load current then becomes:

$$I_o = \frac{V_o - E}{R} \quad (26)$$

The output voltage v_o can be expressed in Fourier series as:

$$v_o = V_o + \sum_{n=1}^{\infty} (a_n \cos n\omega t + b_n \sin n\omega t) \quad (27)$$

Where

$$a_n = \frac{V_m}{\pi} \left\{ \int_{\alpha}^{\pi} \sin \omega t \cos n\omega t d\omega t + \int_{\pi+\alpha}^{2\pi} \sin \omega t \cos n\omega t d\omega t \right\} \quad (28)$$

$$b_n = \frac{1}{\pi} \left\{ \int_{\alpha}^{\pi} V_m \sin \omega t \sin n \omega t d \omega t + \int_{\pi+\alpha}^{2\pi} V_m \sin \omega t \sin n \omega t d \omega t \right\} \quad (29)$$

$$i_o = \sum_{n=1}^{\infty} \frac{1}{Z_n} [a_n \cos(n \omega t - \theta_n) + b_n \sin(n \omega t - \theta_n)] \quad (30)$$

$$Z_n = \sqrt{R^2 + (n \omega L)^2} ; \theta_n = \tan^{-1} \left(\frac{n \omega L}{R} \right)$$

Under steady-state conditions, the armature voltage is:

$$V_a = \frac{V_m}{\pi} (1 + \cos \alpha) \quad (31)$$

The Field circuit voltage is:

$$V_f = \frac{V_m}{\pi} (1 + \cos \alpha_f) \quad (32)$$

From the dc motor operating equations derived earlier, the following equations are produced in conjunction with the rectifier output voltage:

$$I_a = \frac{V_a - E}{R_a} = \frac{V_a - k I_f \omega_m}{R_a} \quad (33)$$

$$I_f = \frac{V_f}{R_f} \quad (34)$$

$$T_a = k I_f I_a = K I_a \quad (35)$$

$$\omega_m = \frac{V_a - I_a R_a}{K} \quad (36)$$

$$\omega_m = \frac{V_a}{K} - \frac{R_a}{(K)^2} T_a \quad (37)$$

III. RESULTS AND DISCUSSIONS

The control equations derived in chapter three of this thesis are plotted here, where,

$P = 1500\text{W}$, $V_a = 200\text{V}$, $I_a = 15\text{A}$, $N_m = 1500\text{rpm}$, $R_a = 6.5\Omega$, $L_a = 25\text{mH}$, $R_f = 100\Omega$, Source voltage, $V_s = 230\text{V}$, $f = 50\text{Hz}$, $J = 0.1\text{kg}\cdot\text{m}^2$, $B = 0.038\text{N}\cdot\text{m}/\text{rad}/\text{s}$.

The armature voltage required to drive the motor at rated speed (157 rad/s) is 200 V, as expressed in equation (23). The plot is shown in Figure 6. Figure 6 is the time response of the motor speed due to step change in the armature voltage.

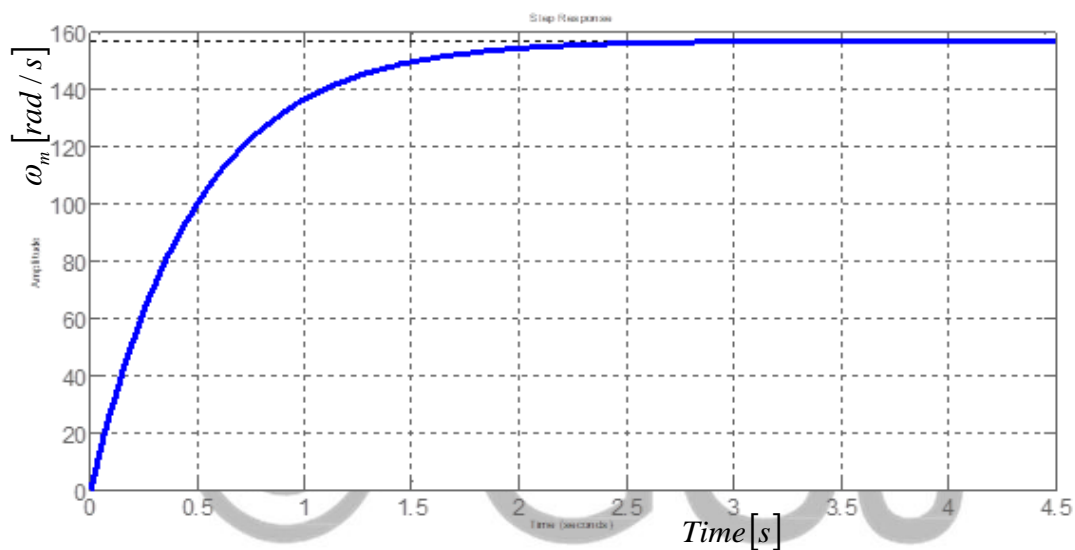


Fig 6 Speed response due to step change in armature voltage.

Application of the rated load of 9.55 N.m, at rated armature voltage of 200 V reduces the motor speed to 110 rad/s as shown in Figure 7 and expressed by equation (24). Figure 7 is the time response of the motor speed due to step change in the load torque. This is the full-load speed.

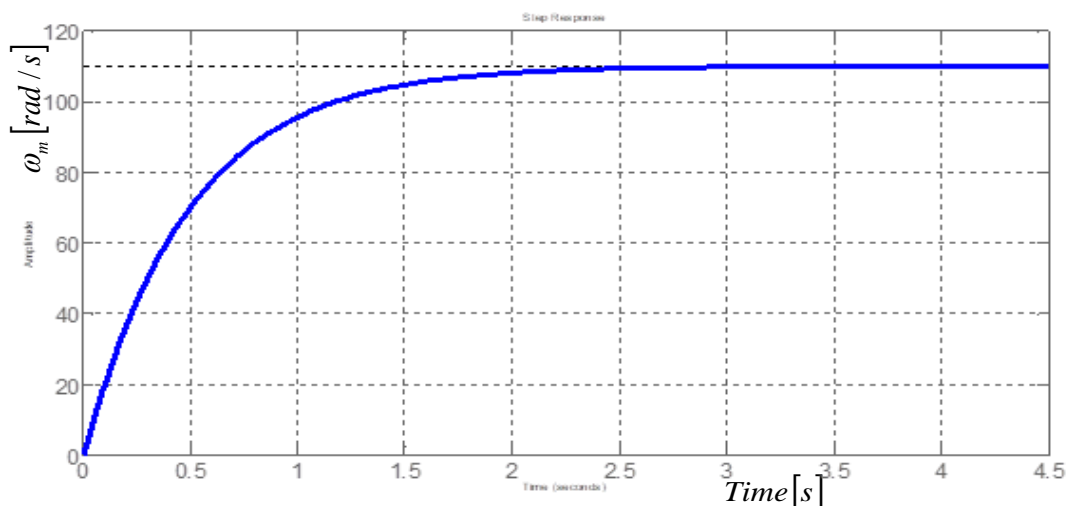


Fig.7 Speed response due to step change in armature voltage and load torque.

The output voltage and output current of the rectifier are expressed in Equations (25), (26), (27) and (30). They are plotted against time as shown in Figure 8. v_o and i_o are the instantaneous quantities, while V_o and I_o are the average values. It can be noticed in Figure 8 that $V_o = 200\text{V}$, $I_o = 15\text{A}$, the rated values of the motor, where $\alpha = 21.3^\circ$.

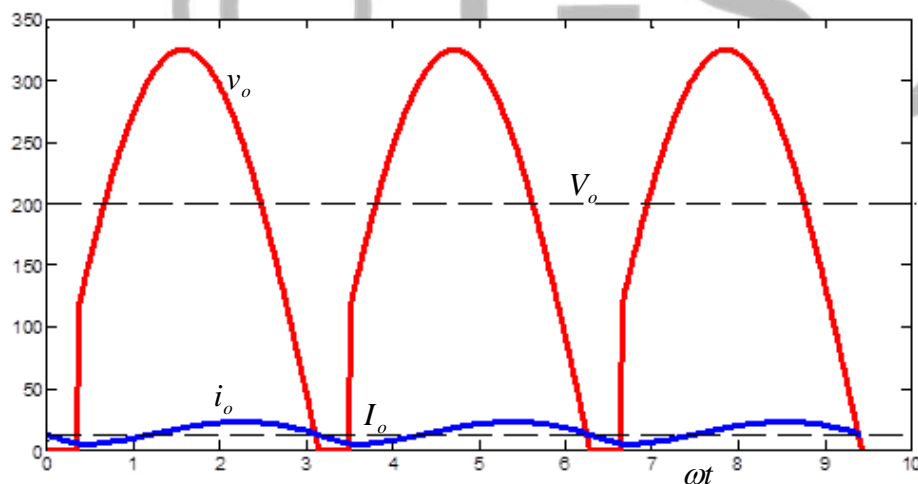


Fig. 8: Output Voltage and Current Waveforms of the armature circuit rectifier

Equation (31) is plotted as shown in Figure 9 which shows how the rectifier output voltage (V_a) varies with the delay angle (α). α is adjustable between 0° and 90° . The maximum armature voltage obtainable is 207 V, at $\alpha = 0^\circ$. At the delay angle of 90° , $V_a = 103\text{V}$. At $\alpha = 21.3^\circ$, the rated voltage of the motor, 200 V is applied across the armature.

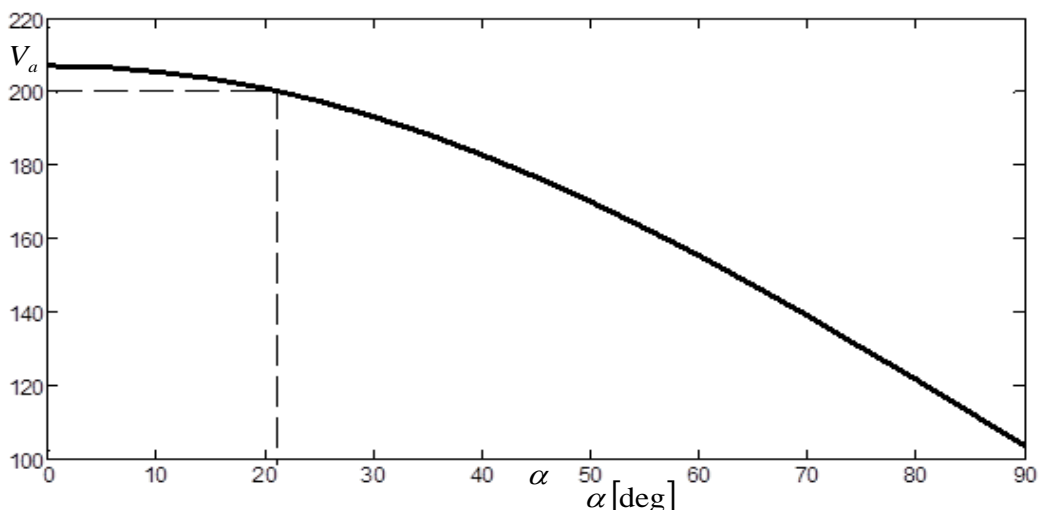


Fig. 9 Variation of armature voltage V_a with delay angle α

Equation (35) shows that the developed torque is directly proportional to the armature current. This is shown in Figure 10. The speed of dc motor changes with the load torque. To maintain a constant speed, the armature voltage or field voltage should be varied continuously by varying the delay angle of ac-dc converters. Thus, an increase in voltage will result in a proportional increase in current, and consequently an increase in developed torque as shown in Figure 10. The figure shows that the torque of 15.52 N.m is developed at the rated current and rated voltage of 15 A and 200 V respectively

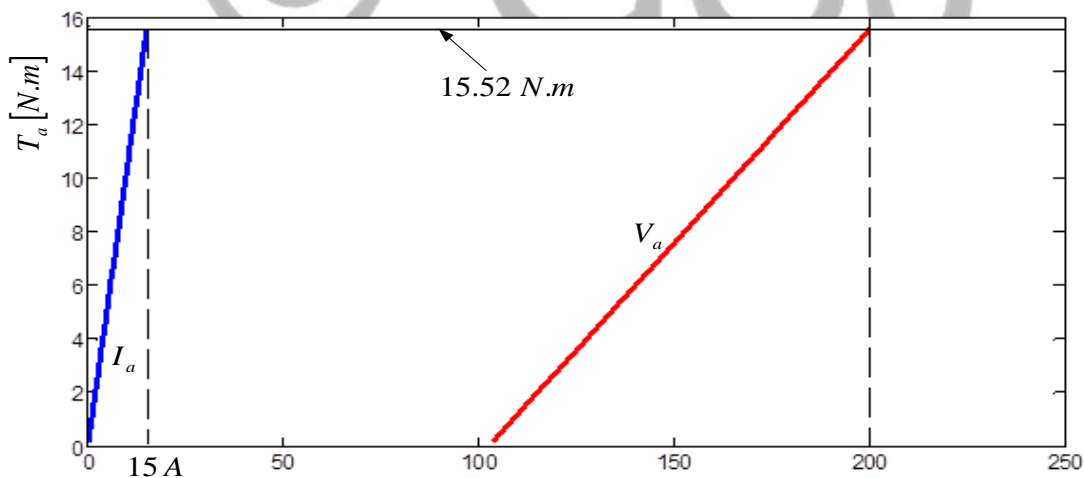


Fig. 10 Plot of developed torque (T_m) versus armature voltage (V_a) and armature current (I_a)

Equation (36) shows that if the armature current increases proportionally with increase in the armature voltage, the speed remains constant, as shown in Figure 11, and since the current is increasing, the developed torque also increases in accordance with equation (3.45).

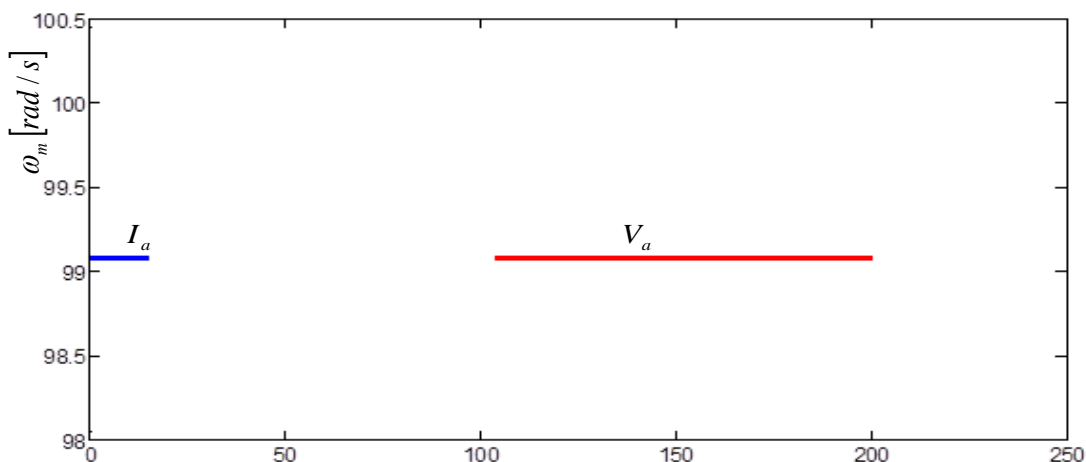


Fig. 11: Plot of motor speed versus armature voltage and current

In equation (37), the torque is kept constant at rated value, while the armature voltage is varied to control the speed. Figure 12 shows how the speed (ω_m) of the motor varies with the armature voltage (V_a). The speed increases as the armature voltage increases.

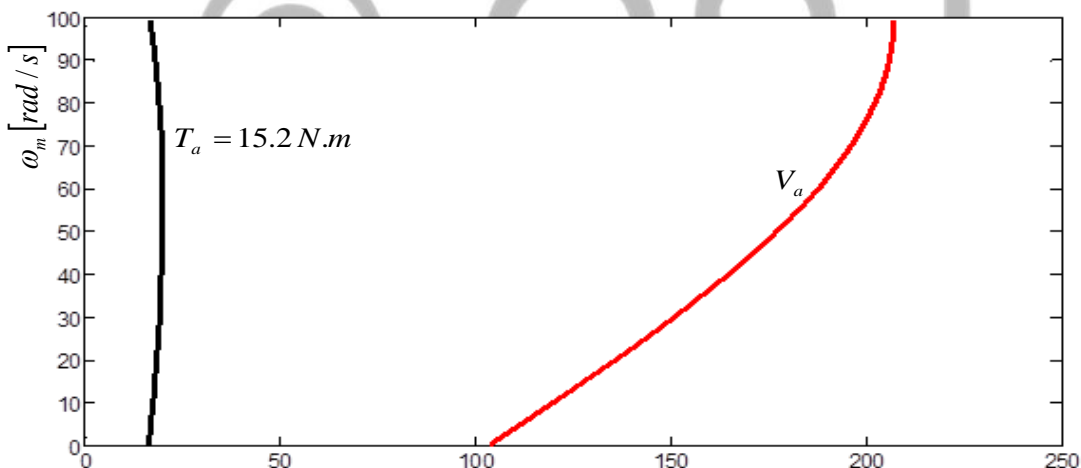


Fig. 12: Speed (ω_m) versus armature voltage (V_a) and developed torque (T_a)

When the motor is to run above base speed, field control is applied. The base speed is the speed obtained at the rated armature voltage. Speed is increased by decreasing the field voltage, while the armature voltage is maintained constant at rated value. Decreasing V_f leads to decrease in I_f . Figure 13 shows the variation of speed with the field excitation. It can be observed that the speed (ω_m) is inversely proportional to the field current (I_f), as indicated in equation (36). The speed is increased by decreasing the field current (I_f).

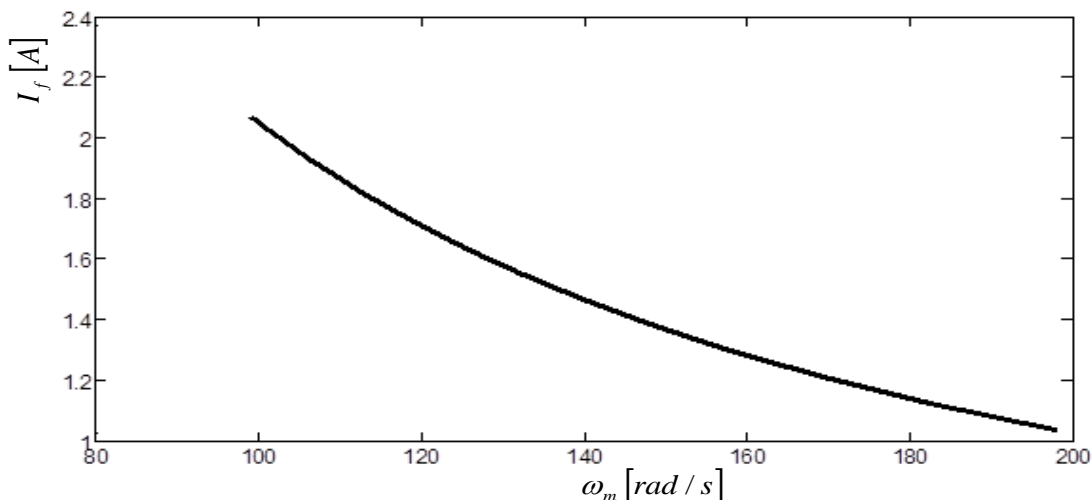


Fig. 13: Plot of speed versus field current

Figure 14 is the plot of torque versus field current. It shows that the developed torque (T_a) increases with increase in field current (I_f), as expressed in equation (35).

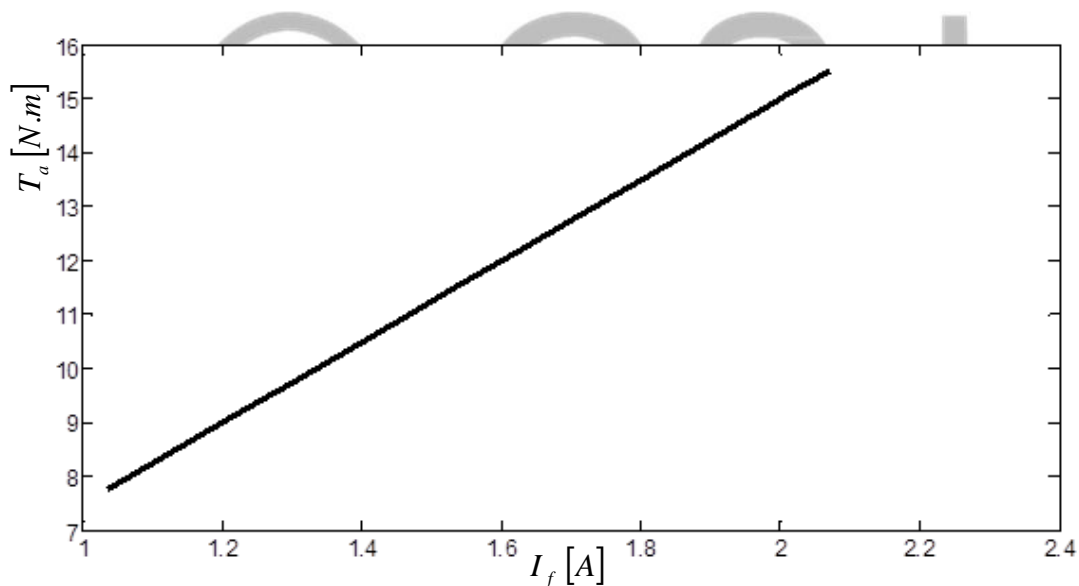


Fig. 14: Plot of Torque Versus Field Current

The speed-torque characteristic of the motor is shown in Figure 15. It shows that the speed (ω_m) decreases as torque (T_a) increases or vice versa. This relation is expressed by equation (37).

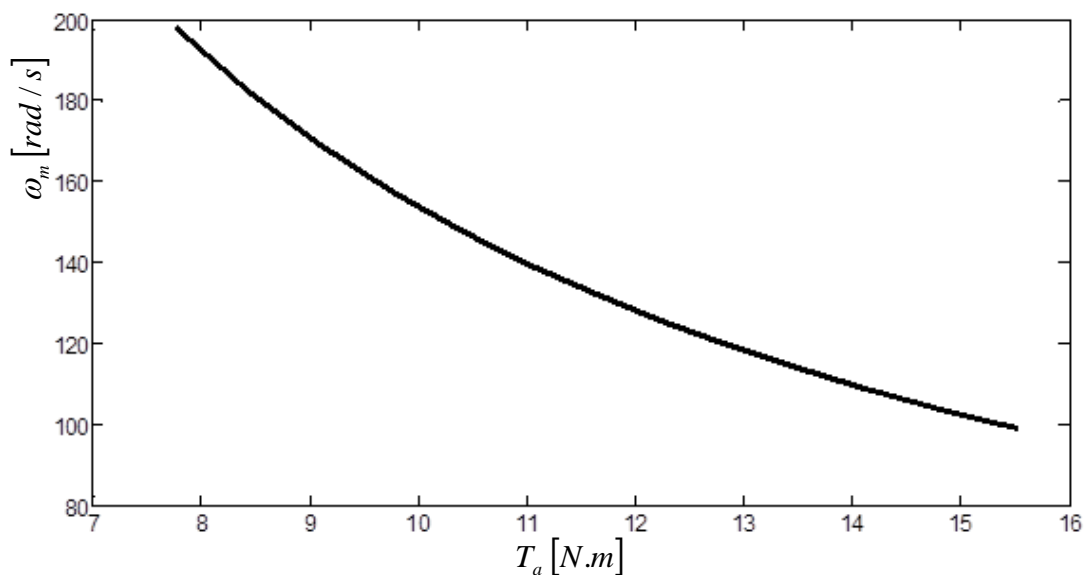


Fig. 15 Plot of Torque Versus Speed.

IV. CONCLUSION

In this thesis the features and operations of different types of dc motor have been presented. The speed of dc motors is controlled by various mean. This thesis has examined speed control of separately excited dc motor by single phase semi-controlled rectifier. This rectifier provides one-quadrant operation, where both the output voltage and current have positive polarity. The principle of operation of the the rectifier has been analyzed. The output current of the rectifier is continuous by virtue of the free-wheeling diode. It has been shown in this thesis that the armature voltage is varied to obtain a speed up to base speed, while the torque is maintained constant. For the base speed to be exceeded, the field current has to be decreased from the rated value. As the torque requirement is decreased, the speed increases. Closed-loop feedback systems are prevalent in industrial drive systems since they enhance accuracy and fast dynamic response. Therefore, closed-loop control was also investigated in this thesis. The transfer functions, which represent the dynamic behavior of the motor with respect to the two possible inputs (armature voltage and load torque) were obtained. The steady-state responses of the motor were obtained by combining the individual response due to the armature voltage and the load torque. Ultimately, it has been shown that, with both armature voltage control and field voltage control the speed and the torque of the motor can be adjusted.

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