



## EFFECT OF SILENCER WEIGHT ON NOISE REDUCTION IN AUTOMOBILE ENGINES

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### Abstract

Effects of the weight function on noise reduction in automobile engines is studied using (2+1) dimensional partial differential wave equation with damping function. The study establishes a design framework for the development and analysis of an active noise control system which can be applied to any vibrio-acoustic system mathematically, the acoustic pressure (unwanted noise) will be modelled by the wave equation defined on a two- dimensional domain (acoustic chamber). The acoustic and structural systems model equation derived is solved using finite difference method with a forward in time central in space (FTCS) numerical scheme. The effect of silencer weight on noise reduction from automobiles was then investigated.

**Key words:** (2+1) Wave equation, damping parameter, silencer weight, Finite Difference Method [Forward in Time Central in Space Scheme]

**Mathematic Subject Classification:** Primary 65N30, 65M12, 65M06: Secondary 65D05, 65M22, 65M60

### INTRODUCTION AND LITERATURE REVIEW

In the field of acoustics, noise is defined as an unpleasant or disliked sound. This definition is straightforward, but the difference between sound and noise is by no means precise. For example, in the opinion of some (older) people the sound of modern music is the equivalent of noise. On the other hand, few would say that the sound produced by passing traffic or a vacuum cleaner is pleasant as cited by Araujo and Madeira [1]. Noise problems have been around for a long time but the development of noise reduction technology is increasingly stimulated for several reasons. One reason is the realization that long-term exposure to high sound levels leads to hearing damage or even hearing loss. Furthermore, because of the tendency towards lightweight design noise problems arise more often. There are various methods for tackling noise problems. In the present work, a relatively new method is considered which is based on the application of active control techniques. Emphasis is on the development of analysis tools, the validation of these tools and their application for designing control systems for noise reduction. Sales in emerging countries such as China and India are thriving resulting in a global increase in the number of vehicles on the market. As of 2009, the total number of road vehicles worldwide was estimated to over a billion units a figure which may have doubled by 2016. Existing vehicle concepts are constantly being improved on in terms fuel consumption, safety, noise and other important characteristics but achieving drastic and efficient improvements in the future cannot be done without relying on multi-functional studies. This necessitates a change in the vehicle design process; future vehicles have to be thought of in terms of multi-functional systems in a larger extent, rather than in terms of individual sub-systems. Liu [2] proposed the spectral properties of a

linear two-dimensional hybrid system arising in the development of these new technologies for noise reduction in the interior of a cavity (plane, car, etc.) in a series of works. The idea of active control of sound and vibration is not new; Gao et al [3] patented a technique for controlling sound with additional sound in the context of the control and stabilization of the wave equation in bounded domains, if one characteristic ray escapes to the dissipative region we cannot expect a uniform decay to hold. Indeed, the nature of the coupling between the acoustic and elastic components of the system that allow to build solutions with arbitrarily slow decay rate with the energy distributed in all of the domain and not only along some particular ray of geometrical optics as in Yang et al [4]. Montenegro and Onorati [5] have shown that in various one-dimensional hybrid systems the coupling is such that the damping term is a compact perturbation of the underlying conservative dynamics. This kind of arguments does not apply in our problem; we are in a (2+1) dimensional space. Actually, Kalantarov et al [6] proved that, in a similar system the difference between the semi group generated by the dissipative system and the one generated by the corresponding conservative system is not compact. Let us mention that a similar problem in which Neumann boundary conditions is considered for the string was studied in details as in Farshbaf-Shaker et al [7]. Optimal Neumann boundary control of a vibrating string with uncertain initial data and probabilistic terminal constraints. From the mathematical point of view, this case is easier since it allows us to separate the variables and to obtain explicit information about the Eigen values and Eigen functions of the system. Issues related to the asymptotic behavior of a hybrid system with two types of vibrations of different nature were introduced in Hedrih [8] research papers. However, there are some important differences between these two models; Wu et al [9] highlights that a flexible damped beam instead of a flexible string occupies the flexible part of the boundary. Qin et al [10] showed that examples of absorption-based techniques are sound absorbing materials such as glass wool and foam and (coupled) tube resonators. Gao et al [3] suggested that sound can be reflected with single or double wall panels for instance as shielding for noisy machinery. Passive methods provide an adequate solution to many noise problems, but have the drawback that they tend to be more attractive for the higher frequencies ( $> 1000$  Hz). At low frequencies, passive methods often lead to an unacceptable increase in mass and volume. In contrast to passive methods, active control methods rely on an external energy source. Active control systems can take many forms but such a system typically consists of sensors to detect a response, an electronic controller to suitably manipulate the sensor signals and actuators to influence the response. Wickman [11] found out that active control of noise is mainly suited for the low frequency range where passive methods are less attractive. The complimentary use of active and passive methods is thus an attractive solution to noise (or vibration) problems. The subject of this proposal is an active control method for reducing the noise produced by vibrating structures (automobiles). More precisely, the objective is to investigate the vibration of the structure in such a way that the sound radiation is minimized. From the review above, it can be inferred that studies on problem of the active control of noise generated in acoustic cavities by means of the vibrations of their flexible walls has received considerable attention using numerical methods. It appears that minimal work has been reported on investigation of stabilization and solution of mathematical noise control model of a (2+1) dimensional wave equation using Finite Difference Method. Introduction For this reason, we will address this problem for the model equation used (wave equation). However, there exists a gap for further investigation of noise control

numerically with use of Finite Difference Method.

### THE (2+1) DIMENSIONAL WAVE EQUATION WITH DAMPING PARAMETER

The geometry of the problem shall be investigated using a (2+1) dimensional partial differential model Wave equation with damping parameter. The acoustic medium in the silencer plate is described by the partial differential wave equation in the variable  $u$  is as follows:

$$\frac{\partial^2 u}{\partial t^2} = c^2 \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - d_i \frac{\partial u}{\partial t} + f(w) \tag{1}$$

The two-dimensional partial differential wave equation with damping effect above is of order 2, degree 1, where  $x$  and  $y$  are the spatial variables while  $t$  is a temporal variable of time. The solution  $u = (x, y, t)$  represents the silencer noise,  $u$ - dependent variable and  $(x, y, t)$  independent variables. The constant  $c^2$  denotes the speed of sound in the fluid (speed = 340m/sec). The non-negative constants  $d_i$  represent damping coefficients (viscous damping and boundary damping) with and  $f(w)$  representing weight function of the silencer plate.

### FINITE DIFFERENCE METHOD

Numerical solution of a mathematical equation gives approximate solution to the problem, for example, the unknown variable is solved at discrete points in space and time. The Finite Difference Method (FDM) basically involves replacing the partial derivatives occurring in the PDE as well as the boundary or initial conditions with their corresponding finite difference approximations and then solving the resulting linear algebraic system of equations by the method of Forward in Time, Central in Space (FTCS). An iteration procedure has to be developed which considers the non- linear character of the equation.

### DISCRETIZATION OF (2+1) DIMENSIONAL WAVE EQUATION

The FTCS scheme in Equation is used to find the effects of varying weight function  $f(w)$  of the exhaust pipe in the noise reduction. Discretizing Equation (1) gives;

$$\left( \frac{u_{i,j}^{n+1} - 2u_{i,j}^n + u_{i,j}^{n-1}}{(\Delta t)^2} \right) = \left[ \frac{u_{i+1,j}^n - 2u_{i,j}^n + u_{i-1,j}^n}{(\Delta x)^2} + \frac{u_{i,j+1}^n - 2u_{i,j}^n + u_{i,j-1}^n}{(\Delta y)^2} - \frac{u_{i,j}^{n+1} - u_{i,j}^n}{\Delta t} + f(w) \right] \tag{2}$$

Rearranging Equation (2) so that values of  $U$  at time  $n+1$  are on the left, and values of  $U$  at time  $n$  are on the right and letting  $r = \frac{\Delta t}{(\Delta x)^2} = \frac{\Delta t}{(\Delta y)^2}$ ,  $\mu = \frac{1}{\Delta t}$  and  $\Delta x = \Delta y$  on a square mesh into

(2) gives,

$$U_{i,j}^{n+1} = \frac{0.3125}{0.0625+d_i} U_{i+1,j}^n - \frac{1.125+d_i}{0.0625+d_i} U_{i,j}^n + \frac{0.3125}{0.0625+d_i} U_{i-1,j}^n + \frac{0.3125}{0.0625+d_i} U_{i,j+1}^n + \frac{0.3125}{0.0625+d_i} U_{i,j-1}^n + \frac{f(w)}{0.0625+d_i} \tag{3}$$

Thus, equation (3) is one equation in a system of equations for the values of  $U$  at the internal nodes of the spatial mesh  $i = 1,2,3,\dots,N-1$ . We write equation (3) such that values of  $U$  from time step  $n+1$  and time step  $n^{\text{th}}$  appear on the left and right hand side respectively. Equation (3) is used to predict the values of  $U$  at time  $n+1$ , so all values of  $U$  at time  $n$  on right hand side are assumed to be known. We assume the sound energy produced is moving along  $x$  - axis. If  $i$  is varied as  $i = 1, 2, 3...10$ , and we fix  $n = 0$  and  $j = 1$  while we let  $\Delta t = 0.0125$ ,  $\Delta x$

$= \Delta y = 0.2, \implies, \mu = 0.0625, r = 0.3125$  in Equation (4.2), then the systems of 10 linear algebraic equations obtained for FTCS scheme are,

$$\left. \begin{aligned}
 i = 1 : U_{1,1}^1 &= \frac{0.3125}{0.0625+d_i} U_{2,1}^0 - \frac{1.125+d_i}{0.0625+d_i} U_{1,1}^0 + \frac{0.3125}{0.0625+d_i} U_{0,1}^0 + \frac{0.3125}{0.0625+d_i} U_{1,2}^0 + \frac{0.3125}{0.0625+d_i} U_{1,0}^0 + \frac{f(w)}{0.0625+d_i} \\
 i = 2 : U_{2,1}^1 &= \frac{0.3125}{0.0625+d_i} U_{3,1}^0 - \frac{1.125+d_i}{0.0625+d_i} U_{2,1}^0 + \frac{0.3125}{0.0625+d_i} U_{1,1}^0 + \frac{0.3125}{0.0625+d_i} U_{2,2}^0 + \frac{0.3125}{0.0625+d_i} U_{2,0}^0 + \frac{f(w)}{0.0625+d_i} \\
 i = 3 : U_{3,1}^1 &= \frac{0.3125}{0.0625+d_i} U_{4,1}^0 - \frac{1.125+d_i}{0.0625+d_i} U_{3,1}^0 + \frac{0.3125}{0.0625+d_i} U_{2,1}^0 + \frac{0.3125}{0.0625+d_i} U_{3,2}^0 + \frac{0.3125}{0.0625+d_i} U_{3,0}^0 + \frac{f(w)}{0.0625+d_i} \\
 i = 4 : U_{4,1}^1 &= \frac{0.3125}{0.0625+d_i} U_{5,1}^0 - \frac{1.125+d_i}{0.0625+d_i} U_{4,1}^0 + \frac{0.3125}{0.0625+d_i} U_{3,1}^0 + \frac{0.3125}{0.0625+d_i} U_{4,2}^0 + \frac{0.3125}{0.0625+d_i} U_{4,0}^0 + \frac{f(w)}{0.0625+d_i} \\
 i = 5 : U_{5,1}^1 &= \frac{0.3125}{0.0625+d_i} U_{6,1}^0 - \frac{1.125+d_i}{0.0625+d_i} U_{5,1}^0 + \frac{0.3125}{0.0625+d_i} U_{4,1}^0 + \frac{0.3125}{0.0625+d_i} U_{5,2}^0 + \frac{0.3125}{0.0625+d_i} U_{5,0}^0 + \frac{f(w)}{0.0625+d_i} \\
 i = 6 : U_{6,1}^1 &= \frac{0.3125}{0.0625+d_i} U_{7,1}^0 - \frac{1.125+d_i}{0.0625+d_i} U_{6,1}^0 + \frac{0.3125}{0.0625+d_i} U_{5,1}^0 + \frac{0.3125}{0.0625+d_i} U_{6,2}^0 + \frac{0.3125}{0.0625+d_i} U_{6,0}^0 + \frac{f(w)}{0.0625+d_i} \\
 i = 7 : U_{7,1}^1 &= \frac{0.3125}{0.0625+d_i} U_{8,1}^0 - \frac{1.125+d_i}{0.0625+d_i} U_{7,1}^0 + \frac{0.3125}{0.0625+d_i} U_{6,1}^0 + \frac{0.3125}{0.0625+d_i} U_{7,2}^0 + \frac{0.3125}{0.0625+d_i} U_{7,0}^0 + \frac{f(w)}{0.0625+d_i} \\
 i = 8 : U_{8,1}^1 &= \frac{0.3125}{0.0625+d_i} U_{9,1}^0 - \frac{1.125+d_i}{0.0625+d_i} U_{8,1}^0 + \frac{0.3125}{0.0625+d_i} U_{7,1}^0 + \frac{0.3125}{0.0625+d_i} U_{7,2}^0 + \frac{0.3125}{0.0625+d_i} U_{8,0}^0 + \frac{f(w)}{0.0625+d_i} \\
 i = 9 : U_{9,1}^1 &= \frac{0.3125}{0.0625+d_i} U_{10,1}^0 - \frac{1.125+d_i}{0.0625+d_i} U_{9,1}^0 + \frac{0.3125}{0.0625+d_i} U_{8,1}^0 + \frac{0.3125}{0.0625+d_i} U_{8,2}^0 + \frac{0.3125}{0.0625+d_i} U_{9,0}^0 + \frac{f(w)}{0.0625+d_i} \\
 i = 10 : U_{10,1}^1 &= \frac{0.3125}{0.0625+d_i} U_{11,1}^0 - \frac{1.125+d_i}{0.0625+d_i} U_{10,1}^0 + \frac{0.3125}{0.0625+d_i} U_{9,1}^0 + \frac{0.3125}{0.0625+d_i} U_{9,2}^0 + \frac{0.3125}{0.0625+d_i} U_{10,0}^0 + \frac{f(w)}{0.0625+d_i}
 \end{aligned} \right\} (4)$$

We take the interior node inside the mesh and along the x and y axes to be the known values as the initial and boundary conditions taken at  $t = 0, x = 0$  and  $y = 0$  respectively. At lower and upper boundary i.e the minimum and maximum values of length of silencer plates is assumed to be between  $x = 0$  and  $x = 10$  respectively while the width between silencer plates is assumed to be between  $y = 0$  and  $y = 2$  respectively in the study. Thus, this gives the following initial and boundary conditions for the weight function of the silencer plate used with known values on the right hand side of the algebraic equations (4). These conditions include;

$$u(x, y, 0) = 1 \tag{5}$$

$$u(0, y, t) = 0, u(10, y, t) = 0, 0 \leq x \leq 10, u(x, 0, t) = 0, u(x, 2, t) = 0, 0 \leq y \leq 2 \tag{6}$$

Equations (5) and (6) are initial and boundary conditions respectively used in the set of the algebraic equations (4) for the known values on the right-hand side of the algebraic equations. The finite difference equations obtained at any space node, say,  $i$  at the time level  $n^{\text{th}}$  left hand side has only three known coefficients involving space nodes at  $i, i-1$  and  $i+1$ , and unknown at  $(n+1)^{\text{th}}$  on the right hand side of the algebraic equations (4). In matrix notation, the 10 algebraic equations in (4) can be expressed in form of  $\mathbf{C}\mathbf{U} = \mathbf{D}$  where  $\mathbf{U}$  is the known vector of order 10 at any time level  $n^{\text{th}}$ .  $\mathbf{D}$  is the unknown vector of order 10 which has the value of  $U$  at the  $(n+1)^{\text{th}}$  time level and  $\mathbf{C}$  is the coefficient square matrix of order  $10 \times 10$ , which is a tridiagonal structure. Then the 10 systems of equations in (4) can be written with 10 unknowns in matrix-vector form as;

$$\begin{pmatrix} U_{1,1}^1 \\ U_{2,1}^1 \\ U_{3,1}^1 \\ U_{4,1}^1 \\ U_{5,1}^1 \\ U_{6,1}^1 \\ U_{7,1}^1 \\ U_{8,1}^1 \\ U_{9,1}^1 \\ U_{10,1}^1 \end{pmatrix} = \begin{pmatrix} \left(\frac{1.125+d_i}{0.0625+d_i}\right) & \left(\frac{0.3125}{0.0625+d_i}\right) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \left(\frac{0.3125}{0.0625+d_i}\right) & \left(\frac{1.125+d_i}{0.0625+d_i}\right) & \left(\frac{0.3125}{0.0625+d_i}\right) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \left(\frac{0.3125}{0.0625+d_i}\right) & \left(\frac{1.125+d_i}{0.0625+d_i}\right) & \left(\frac{0.3125}{0.0625+d_i}\right) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \left(\frac{0.3125}{0.0625+d_i}\right) & \left(\frac{1.125+d_i}{0.0625+d_i}\right) & \left(\frac{0.3125}{0.0625+d_i}\right) & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \left(\frac{0.3125}{0.0625+d_i}\right) & \left(\frac{1.125+d_i}{0.0625+d_i}\right) & \left(\frac{0.3125}{0.0625+d_i}\right) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \left(\frac{0.3125}{0.0625+d_i}\right) & \left(\frac{1.125+d_i}{0.0625+d_i}\right) & \left(\frac{0.3125}{0.0625+d_i}\right) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \left(\frac{0.3125}{0.0625+d_i}\right) & \left(\frac{1.125+d_i}{0.0625+d_i}\right) & \left(\frac{0.3125}{0.0625+d_i}\right) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \left(\frac{0.3125}{0.0625+d_i}\right) & \left(\frac{1.125+d_i}{0.0625+d_i}\right) & \left(\frac{0.3125}{0.0625+d_i}\right) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \left(\frac{0.3125}{0.0625+d_i}\right) & \left(\frac{1.125+d_i}{0.0625+d_i}\right) & \left(\frac{0.3125}{0.0625+d_i}\right) \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \left(\frac{0.3125}{0.0625+d_i}\right) & \left(\frac{1.125+d_i}{0.0625+d_i}\right) \end{pmatrix} \begin{pmatrix} U_{1,1}^0 \\ U_{2,1}^0 \\ U_{3,1}^0 \\ U_{4,1}^0 \\ U_{5,1}^0 \\ U_{6,1}^0 \\ U_{7,1}^0 \\ U_{8,1}^0 \\ U_{9,1}^0 \\ U_{10,1}^0 \end{pmatrix} + \begin{pmatrix} \frac{f(w)}{0.0625+d_i} \\ \frac{f(w)}{0.0625+d_i} \end{pmatrix} \quad (7)$$

### EFFECTS OF WEIGHT FUNCTION IN NOISE REDUCTION

The effects of silencer plate- weight function in noise reduction in automobile engines is under studies. The damping parameter  $d_i$  is held constant and taken as  $d_i = 8$  in (7), we get the matrix equation shown in Equation (7)

$$\begin{pmatrix} U_{1,1}^1 \\ U_{2,1}^1 \\ U_{3,1}^1 \\ U_{4,1}^1 \\ U_{5,1}^1 \\ U_{6,1}^1 \\ U_{7,1}^1 \\ U_{8,1}^1 \\ U_{9,1}^1 \\ U_{10,1}^1 \end{pmatrix} = \begin{pmatrix} 1.0069 & 0.0344 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.0344 & 1.0069 & 0.0344 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.0344 & 1.0069 & 0.0344 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.0344 & 1.0069 & 0.0344 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.0344 & 1.0069 & 0.0344 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.0344 & 1.0069 & 0.0344 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.0344 & 1.0069 & 0.0344 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.0344 & 1.0069 & 0.0344 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0344 & 1.0069 & 0.0344 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0344 & 1.0069 \end{pmatrix} \begin{pmatrix} U_{1,1}^0 \\ U_{2,1}^0 \\ U_{3,1}^0 \\ U_{4,1}^0 \\ U_{5,1}^0 \\ U_{6,1}^0 \\ U_{7,1}^0 \\ U_{8,1}^0 \\ U_{9,1}^0 \\ U_{10,1}^0 \end{pmatrix} + \begin{pmatrix} \frac{f(w)}{8.0625} \\ \frac{f(w)}{8.0625} \end{pmatrix} \quad (8)$$

The silencer plate weight function  $f(w)$  is varied with four different arbitrary values i.e  $f(w) = 10N, 20N, 30N, 40N$  in (8). With each value substituted at a time in (8), the matrix equation has been solved using MATLAB and a set of four different values of the noise intensity obtained as shown in table 1.

Table 1: Effects of varying the Weight Function of the Silencer Plate on Noise Reduction

X	f(w) = 40N	f(w) = 30N	f(w) = 20N	f(w) = 10N
1	45.3504	23.7676	16.1076	12.1822
2	2.2908	1.7607	1.3278	1.0564
3	4.2521	2.2849	1.5663	1.1921
4	4.1628	2.2724	1.5624	1.1905
5	4.1628	2.2727	1.5625	1.1905
6	4.1667	2.2727	1.5625	1.1905
7	4.1666	2.2727	1.5625	1.1905
8	4.1671	2.2727	1.5625	1.1905
9	4.1580	2.2714	1.5621	1.1903
10	4.3565	2.3269	1.5877	1.2050

Table 1 shows that at different values of  $x$ , the respective values of sound wave frequency have been obtained when weight functions  $f(w)$  is varied at  $f(w) = 10N, 20N, 30,$  and  $40N$ . It is noted that sound wave particles flow at a higher kinetic energy at distance near source of sound and the intensity decrease a distance away from source of sound. This implies that collision of sound wave particles is higher at distance of the plates near source of sound thus causing high vibration. As the distance increases away from source of sound, kinetic energy decrease due to increased viscosity hence resulting to reduction of vibration of the particles. This means that vibration of particles decrease with decrease in weight function of the silencer plate. For instance, at  $x = 4$  the values of sound wave frequency decrease from 1.5624 to 1.1905 as weight function decrease from  $f(w) = 20N$  to  $f(w) = 10N$  respectively. It is noted that sound wave frequency changes in the sameway for other values of  $x$ . This result indicates that sound wave frequency at any value of  $x$  decreases with decrease in weight function of the silencer plate and vise versa.

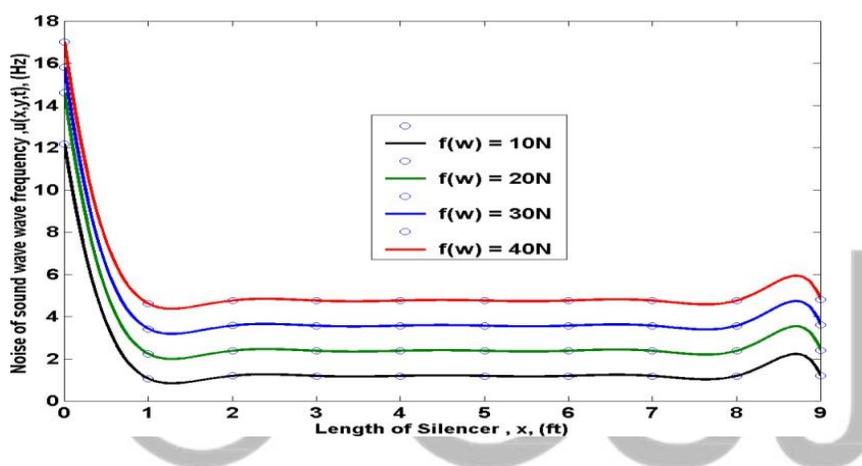


Fig 1: Graph of varied Weight Function  $f(w)$  of the Silencer Plate on noise reduction

Figure 1 shows the resulting sound wave frequency at every respective point of the varied Weight function;  $f(w) = 10N, f(w) = 20N, f(w) = 30N$  and  $f(w) = 40N$  at constant damping parameter  $d_i = 8$  of the silencer plate. It is seen that the sound wave frequency at the source is 12Hz for weight 10N, 15Hz for 20N, 16Hz for 30N and 17Hz for 40N respectively. It is clearly seen that the intensity of sound wave produced from the automobile engine is greatly influenced by the weight of the silencer plate (exhaust pipe). From the results, it is evident that the silencer plate should be made of a very light material so as to help minimize the ultimate sound energy that is being emitted to the environment. For instance, the frequency of the sound wave has steadily risen with increase in the weight function of the silencer plate and with the least weight resulting to least sound wave frequency.

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