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EXPERIMENTS ON OPTIMAL LATIN HYPERCUBE DESIGNS REGARDING EUCLIDEAN MEASURE

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Abstract**:** *A Latin Hypercube Design (LHD) is a statistical design of experiments, which was first defined in 1979. An LHD of k-factors (dimensions) with N design points,* $x_i = (x_{i1}, x_{i2}, \cdots, x_{ik}) : i = 0, 1, \ldots, N-1$*, is given by a* $N\times k$*- matrix (i.e. a matrix with N rows and k columns)* ***X****, where each column of* ***X*** *consists of a permutation of integers* $0, 1, \cdots, N-1$ *(note that each factor range is normalized to the interval* $[0, N-1]$*) so that for each dimension j all* x_{ij} *,* $i = 0, 1, \cdots, N-1$ *are distinct. We will refer to each row of* ***X*** *as a (discrete) design point and each column of* ***X*** *as a factor (parameter) of the design points. We can represent* ***X*** *as follows:*

$$\mathbf{X}=\begin{pmatrix} x_0 \\ \vdots \\ x_{N-1} \end{pmatrix}=\begin{pmatrix} x_{01} & \cdots & x_{0k} \\ \vdots & \cdots & \vdots \\ x_{(N-1)1} & \cdots & x_{(N-1)k} \end{pmatrix}$$

such that for each $j \in \{1, 2, \cdots, k\}$ *and for all* $p, q \in \{0, 1, \cdots, N-1\}$ *with* $p \neq q$*;* $xpj \neq xqj$ *holds. let* $D = \{d(x_i, x_j) : \leq i < j \leq N\}$*. In order to optimized LHD an optimal criterion need to set for searching through LHDs. We consider the* ***Opt(φ,D₁)*** *criterion. In this publication we will discuss about the optimal maximin LHDs obtained by ILS approach in Euclidean distance measure and comparison with existence literatures. At first, we will display the optimal LHDs to show the performance of ILS approach regarding Euclidean distance measure.*

Keywords: *Latin Hypercube Design, Euclidean distance, Statistical design, Iterated Local Search (ILS).*

1.Experimental Results and Discussion for Euclidean Measure:

At first we will compare the performance of several approaches available in the literature regarding maximin LHDs namely maximin LHDs obtained by ILS approach [Grosso et al. (2009)] and denoted by MLH-ILS; maximin LHDs obtained by Periodic design (PD) approach Husslage et al. (2006)] and denoted by MLH-PD; maximin LHDs obtained by Simulated Annealing (SA) approach [Husslage et al. (2006)] and denoted by MLH-SA, maximin LHDs obtained by Simulated Annealing (SA_M) approach [Morris and Mitchel (1995)] and denoted by MLH-SA_M and maximin LHDs obtained by Enhanced Stochastic Evolutionary (ESE) algorithm [Husslage et al. (2011)] which is first proposed by Jin et al. (2005) and the designs are denoted by MLH_ESE.

It is noted that the best maximin LHDs are frequently update in the website https://spacefillingdesigns.nl/ (2008) and we denote it as Best-Web. It will be worthwhile to mention here that we compare maximin LHDs of website during 2009 rather than recent update value (like 2016). We do so because at 2009 the website is updated by taking the values obtained by ILS approach [Grosso et al. (2009)] and accept few values MLH_ILS values reveal best till to date. For the comparison study we consider all designs with $\{(k,N) : k = 3,\ldots, 0 ; N = ,\ldots, 00\}$. In the tables the head line of each design is shorted as MLH_ESE to ESE and so on. The experimental results are reputed in the Table 1.1 and Table 1.2.

Table 1.1: Comparison among PD, SA, ESE and ILS approaches regarding maximin LHDs in Euclidean distance measure for *k*=3-6

	k=3				*k*=4				*k*=5				*k*=6			
N	PD	SA	ESE	**ILS**	PD	SA	ESE	**ILS**	PD	SA	ESE	**ILS**	PD	SA	ESE	**ILS**
2	3	3	3	**3**	4	4	4	4	5	5	5	5	6	6	6	6
3	3	6	6	**6**	4	7	7	7	5	8	8	8	6	12	12	12
4	6	6	6	**6**	12	12	12	12	11	14	14	14	15	20	20	20
5	6	11	11	**11**	12	15	15	15	11	24	24	24	15	27	27	27
6	14	14	14	**14**	16	22	22	22	23	32	32	32	28	40	40	40
7	14	17	17	**17**	16	28	28	28	23	40	40	40	28	52	52	52
8	21	21	21	**21**	25	42	42	42	32	50	50	50	42	66	**63**	66
9	21	22	22	**22**	25	42	42	42	39	61	61	61	45	76	**75**	76
10	21	27	27	**27**	36	50	47	50	55	82	82	82	62	91	91	92
11	24	30	30	**30**	39	55	55	55	55	80	80	81	62	108	108	110
12	30	36	36	**36**	46	63	63	63	62	91	91	93	91	136	**136**	139
13	35	41	41	**41**	51	68	70	70	64	101	103	104	91	136	138	140
14	35	42	42	**42**	70	75	77	79	86	112	114	116	104	152	154	160
15	42	48	48	**48**	71	83	87	89	88	124	129	131	111	167	171	175
16	42	50	50	**50**	85	90	93	94	101	136	151	154	130	186	190	194
17	42	53	53	**54**	85	97	99	103	113	150	158	159	131	203	208	214
18	50	56	56	**57**	94	103	108	111	123	162	170	172	155	223	231	241
19	57	59	59	**62**	94	113	119	122	136	174	184	189	169	241	256	263
20	57	62	65	**66**	106	123	130	137	139	184	206	206	210	260	279	285
21	65	66	68	**69**	116	127	145	149	165	201	223	229	210	283	302	306
22	69	69	72	**76**	117	137	150	151	174	215	235	242	223	304	325	338
23	72	74	75	**77**	130	146	159	161	178	224	250	251	236	324	348	358
24	76	78	81	**83**	138	154	170	170	201	242	266	269	258	343	374	378
25	91	81	**86**	**86**	156	162	178	181	205	255	285	286	286	368	400	408
26	91	86	86	**86**	156	171	188	189	226	269	302	306	296	387	426	439
27	91	90	**90**	**90**	157	178	198	198	238	287	310	326	310	410	447	474
28	94	94	**94**	**94**	174	188	210	212	258	302	331	349	339	427	479	494
29	94	98	**101**	**101**	174	196	221	219	269	322	349	373	346	452	507	517
30	105	102	**105**	**105**	194	209	233	230	310	335	367	403	390	473	531	545
31	107	106	110	**110**	212	215	244	240	310	347	405	406	390	504	563	569
32	114	110	110	**116**	212	228	253	252	341	371	413	418	419	529	587	599
33	114	113	**117**	**120**	215	234	264	267	341	379	426	446	430	548	622	634
34	133	117	**125**	**126**	230	244	273	274	358	403	445	460	470	586	648	668
35	133	122	**126**	**129**	234	255	286	289	366	418	467	482	495	601	683	697
36	133	129	**131**	**136**	250	261	297	298	400	427	486	502	518	631	719	739
37	152	131	**138**	**140**	266	275	309	308	408	454	520	530	528	648	744	775
38	152	134	**142**	**142**	283	279	321	322	415	464	541	557	561	681	788	813
39	152	139	146	**149**	283	290	330	330	439	486	566	575	561	706	816	846
40	155	146	152	**152**	291	301	342	345	492	505	575	590	632	739	876	886
41	162	147	**158**	**155**	293	309	355	354	492	525	596	618	632	776	882	938
42	168	152	**161**	**162**	319	325	367	371	496	543	626	641	670	791	907	988
43	168	157	**171**	**169**	323	329	383	378	520	558	666	664	670	830	947	996
44	186	161	**179**	**178**	331	349	396	393	548	582	680	688	696	862	992	1041
45	186	166	182	**179**	347	362	407	405	565	615	698	706	737	891	996	1065
46	186	169	189	**185**	366	370	421	421	592	615	723	728	797	918	1064	1107
47	186	173	189	**189**	378	378	438	426	611	634	754	762	797	940	1088	1113
48	189	178	**201**	**194**	413	385	450	451	632	673	**763**	782	857	976	1119	1159
49	196	180	203	**201**	415	399	464	463	634	680	803	799	893	1015	1167	1181
50	213	185	**206**	**206**	415	414	478	473	663	699	830	830	893	1042	1203	1218
51	213	189	206	**209**	421	426	490	487	692	727	850	857	917	1067	1230	1258
52	213	198	**217**	**214**	455	429	504	501	709	742	883	874	1003	1100	1274	1292
53	216	200	**219**	**221**	455	447	515	516	716	765	894	901	1003	1136	1340	1340
54	233	213	**209**	**227**	477	454	534	526	760	783	932	935	1019	1171	1359	1392
55	243	214	**230**	**233**	483	477	546	541	760	805	956	966	1082	1198	1421	1432
56	243	216	**230**	235	515	479	558	565	784	830	982	992	1104	1236	1431	1484
57	261	221	**249**	241	515	490	574	570	846	854	1007	1018	1136	1265	1488	1523
58	261	227	**245**	246	539	500	594	591	846	878	1035	1046	1166	1303	1554	1559
59	266	229	**254**	254	544	519	609	607	849	905	1063	1064	1223	1328	1564	1615
60	273	237	**261**	258	568	530	618	622	904	928	1094	1101	1242	1381	1631	1647
61	274	244	**266**	262	620	538	630	641	904	939	1128	1134	1258	1413	1667	1703
62	283	245	**269**	269	620	554	657	645	934	991	1150	1156	1306	1450	1715	1756
63	297	249	**281**	276	620	575	670	666	967	989	1178	1187	1380	1497	1781	1781
64	297	258	**278**	281	625	579	684	678	985	1009	1206	1223	1430	1526	1804	1834
65	314	260	**290**	286	630	582	694	701	997	1035	1216	1239	1430	1565	1868	1884
66	314	269	**299**	294	666	602	718	706	1050	1051	1261	1272	1476	1590	1874	1926
67	314	270	**294**	297	666	614	735	726	1072	1085	1299	1283	1482	1646	1954	1977
68	314	278	**306**	306	685	623	746	738	1087	1119	1330	1360	1538	1664	1983	2014
69	324	280	**306**	310	698	650	765	754	1112	1114	1351	1399	1588	1704	2028	2070
70	325	285	**314**	313	716	658	779	773	1150	1135	1378	1439	1633	1759	2094	2116
71	325	289	**314**	325	716	665	793	795	1150	1187	1413	1416	1644	1783	2141	2168
72	341	296	**314**	326	750	678	810	810	1203	1197	1430	1454	1768	1862	2136	2215
73	350	299	**329**	329	759	688	834	818	1229	1242	1462	1549	1768	1872	2197	2252
74	350	306	**341**	341	767	703	842	845	1229	1269	1512	1562	1774	1910	2291	2299
75	350	310	**341**	345	771	714	867	854	1274	1282	1530	1571	1862	1963	2303	2365
76	363	324	**341**	349	813	750	882	877	1300	1318	1569	1597	1935	2024	2387	2415
77	363	325	**341**	355	823	762	894	890	1308	1331	1591	1631	1947	2051	2433	2456
78	387	337	**371**	362	844	761	910	906	1382	1360	1621	1654	2014	2079	2479	2502
79	387	333	**374**	376	848	788	927	921	1382	1399	1639	1668	2037	2120	2498	2550
80	403	344	**374**	371	873	786	949	943	1395	1430	1691	1690	2037	2152	2554	2597
81	406	338	**381**	381	916	782	963	972	1406	1431	1730	1731	2064	2217	2648	2665
82	406	353	**374**	389	938	825	989	979	1475	1482	1742	1773	2141	2239	2680	2715
83	417	369	**374**	401	940	829	1002	1006	1501	1509	1762	1804	2141	2290	2696	2752
84	426	363	**406**	401	967	838	1021	1015	1534	1510	1818	1825	2229	2325	2790	2803
85	426	369	**413**	406	967	877	1043	1032	1552	1566	1866	1871	2232	2399	2819	2877
86	428	376	**413**	422	967	867	1053	1047	1573	1578	1882	1890	2375	2437	2875	2929
87	428	374	**413**	419	976	877	1073	1062	1598	1589	1934	1922	2375	2476	2913	2988
88	437	374	**434**	426	1050	890	1086	1079	1685	1629	1954	1963	2398	2513	2975	3075
89	443	378	**426**	432	1050	907	1102	1102	1690	1654	1990	2009	2400	2562	3067	3104
90	481	384	**446**	437	1060	940	1134	1116	1710	1696	2027	2032	2516	2633	3104	3134
91	481	393	**434**	450	1089	951	1134	1146	1748	1724	2031	2054	2516	2674	3143	3192
92	481	394	**446**	457	1089	966	1149	1154	1805	1750	2100	2117	2599	2729	3216	3243
93	481	402	**446**	456	1098	962	1171	1169	1813	1795	2130	2122	2604	2726	3283	3305
94	481	405	**470**	461	1124	986	1199	1198	1881	1811	2169	2165	2747	2788	3348	3360
95	481	413	482	474	1135	1010	1219	1223	1901	1846	2206	2202	2747	2817	3335	3455
96	509	414	**486**	475	1261	1023	**1250**	1228	1965	1863	2227	2261	2769	2911	3451	3497
97	515	419	**474**	489	1261	1027	**1258**	1245	1965	1899	2299	2269	2817	2960	3514	3546
98	531	429	**485**	486	1261	1055	1283	1266	1965	1929	2299	2309	2850	3001	3560	3606
99	531	449	**449**	497	1261	1040	1298	1290	2009	1950	2338	2356	2878	3043	3628	3652
100	554	451	**494**	502	1261	1074	1305	1313	2053	1975	2401	2388	3000	3117	3648	3710

Table 1.2: Comparison among PD, SA, ESE and ILS approaches regarding maximin LHDs in Euclidean distance measure for *k*=7 – 10

		k=7			*k*=8			*k*=9			*k*=10		
N	PD	SA	ESE	**ILS**	SA	ESE	ILS	SA	ESE	**ILS**	SA	ESE	**ILS**
2	7	7	7	7	8	8	8	9	9	9	10	10	10
3	7	13	13	13	14	14	14	18	18	18	19	19	19
4	16	21	21	21	26	26	26	28	28	28	33	33	33
5	16	32	32	32	40	40	40	43	43	43	50	50	50
6	29	47	47	47	54	**53**	54	61	61	61	68	68	68
7	31	61	61	61	70	70	71	80	80	81	89	89	90
8	46	79	79	79	91	90	91	101	101	102	114	114	114
9	47	92	92	93	112	112	113	126	126	128	141	142	143
10	68	110	**109**	111	130	131	133	154	154	157	172	**171**	174
11	69	128	129	132	152	152	154	178	178	181	206	206	209
12	95	150	152	155	176	177	181	204	204	209	235	235	240
13	95	174	178	181	202	205	210	232	235	242	267	268	275
14	119	204	**215**	217	228	236	243	265	268	278	298	305	313
15	129	211	220	223	257	273	280	296	309	318	337	347	358
16	155	238	241	249	286	317	326	330	352	358	378	393	406
17	161	256	266	272	312	332	332	367	396	405	415	442	458
18	186	281	291	298	344	361	368	398	451	466	458	496	509
19	195	305	323	326	370	390	398	438	469	472	498	554	569
20	226	332	349	360	403	425	434	472	506	517	542	625	641
21	236	361	380	393	438	463	471	517	548	559	592	650	650
22	270	384	418	425	467	501	508	555	595	614	643	691	704
23	273	410	448	454	501	542	549	596	640	651	685	747	750
24	308	444	481	492	538	585	595	639	690	699	739	800	818
25	350	467	520	531	583	626	637	688	739	752	792	857	875
26	365	499	548	570	612	664	688	726	791	810	854	910	931
27	382	526	585	599	648	712	738	780	840	859	896	976	1002
28	406	561	620	634	693	766	785	826	898	919	953	1041	1061
29	417	593	654	675	733	817	837	876	956	986	1015	1100	1132
30	458	620	691	714	787	849	897	925	1019	1041	1086	1173	1207
31	482	657	728	764	812	900	931	976	1104	1104	1138	1241	1275
32	518	695	778	803	866	966	976	1026	1139	1176	1194	1318	1351
33	537	723	814	844	900	1010	1037	1084	1201	1244	1253	1396	1436
34	561	751	851	891	945	1072	1089	1135	1270	1316	1329	1478	1514
35	586	811	914	934	1002	1113	1151	1190	1326	1398	1398	1555	1595
36	636	831	939	968	1042	1181	1205	1257	1405	1444	1459	1647	1679
37	668	863	976	1012	1079	1236	1272	1300	1477	1505	1516	1721	1761
38	709	923	1028	1055	1127	1286	1328	1367	1534	1577	1597	1790	1852
39	726	938	1084	1094	1192	1344	1397	1434	1609	1640	1665	1870	1987
40	786	970	1122	1148	1224	1416	1459	1489	1675	1728	1742	1946	2101
41	802	1016	1156	1197	1271	1496	1535	1562	1765	1793	1820	2058	2135
42	903	1064	1209	1249	1333	1526	1584	1639	1843	1871	1920	2149	2191
43	903	1112	1256	1301	1377	1597	1635	1683	1905	1957	1973	2224	2279
44	903	1140	1336	1340	1463	1653	1698	1752	1994	2042	2072	2319	2373
45	926	1192	1366	1408	1480	1723	1755	1820	2079	2126	2130	2415	2466
46	985	1243	1408	1448	1548	1794	1819	1906	2155	2220	2208	2507	2568
47	985	1268	1459	1521	1616	1847	1883	1958	2244	2312	2331	2600	2663
48	1054	1325	1531	1578	1658	1924	1957	2017	2336	2383	2387	2732	2760
49	1074	1356	1592	1649	1729	1989	2018	2103	2397	2470	2470	2828	2880
50	1113	1397	1639	1699	1772	2041	2089	2179	2492	2569	2556	2893	2991
51	1161	1450	1662	1744	1855	2132	2152	2243	2566	2637	2639	3006	3090
52	1231	1486	1734	1804	1888	2203	2218	2325	2686	2716	2745	3134	3202
53	1241	1537	1808	1886	1949	2234	2288	2429	2713	2798	2825	3261	3306
54	1288	1577	1856	1932	2006	2356	2383	2473	2805	2884	2892	3339	3412
55	1325	1639	1896	2000	2084	2429	2462	2570	2935	2996	3054	3452	3530
56	1358	1701	2003	2073	2162	2444	2533	2623	3021	3060	3100	3551	3643
57	1479	1721	2024	2098	2194	2554	2620	2704	3119	3162	3215	3651	3767
58	1479	1795	2043	2156	2258	2650	2679	2796	3187	3268	3305	3795	3843
59	1509	1821	2136	2187	2356	2733	2793	2881	3297	3350	3399	3889	3977
60	1577	1899	2232	2277	2393	2796	2873	2939	3420	3446	3500	4090	4109
61	1615	1928	2266	2316	2488	2868	2966	3021	3525	3565	3588	4158	4202
62	1680	2023	2345	2367	2541	2977	3048	3132	3636	3651	3700	4313	4322
63	1680	2035	2376	2417	2607	3056	3160	3215	3690	3760	3767	4355	4445
64	1769	2093	2452	2484	2734	3097	3207	3292	3820	3868	3955	4514	4560
65	1786	2132	2492	2547	2723	3219	3286	3357	3932	3991	4034	4581	4695
66	1857	2180	2543	2606	2841	3279	3418	3474	4004	4088	4143	4769	4818
67	1868	2238	2638	2672	2868	3399	3488	3543	4081	4200	4224	4942	4981
68	1940	2295	2693	2714	2956	3453	3600	3647	4212	4317	4360	4995	5077
69	1965	2351	2746	2794	3075	3520	3704	3716	4317	4400	4455	5127	5221
70	2130	2417	2838	2856	3130	3588	3779	3841	4464	4516	4539	5276	5366
71	2130	2451	2871	2939	3161	3749	3877	3936	4548	4666	4689	5437	5479
72	2177	2503	2960	2992	3220	3810	3962	4027	4666	4758	4812	5556	5625
73	2206	2598	3042	3077	3305	3932	4009	4134	4776	4858	4873	5661	5746
74	2244	2614	3120	3117	3432	3941	4127	4224	4915	4997	5038	5817	5879
75	2295	2703	3157	3230	3513	4073	4213	4298	5006	5141	5171	5937	6015
76	2375	2756	3218	3289	3559	4178	4326	4395	5179	5261	5254	6111	6163
77	2403	2819	3323	3359	3617	4266	4384	4492	5222	5364	5399	6272	6305
78	2505	2870	3387	3432	3684	4390	4491	4577	5385	5543	5489	6384	6449
79	2525	2950	3474	3488	3775	4465	4585	4705	5535	5631	5633	6466	6580
80	2590	2979	3550	3564	3877	4565	4695	4807	5577	5792	5773	6653	6733
81	2642	3086	3619	3638	4001	4679	4721	4888	5748	5922	5901	6780	6842
82	2753	3118	3669	3727	3998	4719	4809	5030	5859	6041	6013	6935	7041
83	2767	3195	3723	3800	4076	4848	4906	5102	5976	6196	6097	7094	7258
84	2838	3227	3870	3883	4183	4920	5006	5222	6119	6357	6273	7256	7362
85	2874	3299	3919	3954	4324	5032	5110	5340	6212	6479	6397	7357	7508
86	3103	3335	3958	4032	4397	5164	5205	5423	6346	6606	6491	7532	7687
87	3103	3450	4095	4119	4474	5225	5302	5538	6469	6761	6622	7639	7837
88	3183	3500	4166	4199	4524	5340	5426	5667	6660	6873	6803	7877	8022
89	3183	3541	4176	4290	4578	5450	5515	5774	6750	7004	6872	7950	8151
90	3190	3661	4308	4362	4699	5576	5608	5832	6901	7152	7040	8128	8325
91	3234	3677	4379	4423	4850	5626	5696	5969	6950	7296	7163	8330	8464
92	3277	3760	4428	4526	4873	5758	5822	6081	7067	7396	7286	8442	8681
93	3361	3811	4512	4574	4984	5832	5925	6231	7342	7446	7488	8601	8828
94	3474	3888	4581	4675	5067	6007	6032	6329	7436	7642	7536	8774	9066
95	3531	3940	4703	4758	5154	6064	6148	6396	7469	7748	7741	8877	9252
96	3639	4070	4808	4862	5220	6222	6227	6516	7645	7926	7777	9146	9445
97	3639	4069	4848	4919	5316	6304	6364	6649	7781	8011	8038	9379	9550
98	3690	4147	4936	5007	5445	6376	6467	6776	7896	8152	8242	9381	9820
99	3731	4214	4999	5117	5477	6448	6571	6912	8023	8258	8344	9617	9998
100	3903	4335	5040	5184	5597	6617	6692	6983	8228	8464	8450	9835	10233

To see the performance of each approaches at a glance, we summarized the above results in Table 1.3. In the Table 1.3, identical means the maximin LHDs obtained by ILS approach is identical compare to the best known results available in the literature whereas worse means the maximin LHDs obtained by ILS approach are worse compare to best known results. Notice that the maximin LHDs obtained by SA_M approach are not reported in the Table 1.1 and Table 1.2 as there are few values available in the literature [Morris and Mitchel (1995)] and all of which are worse with respect to MLH_ILS [Jamali (2009)] as shown in the Table 1.3. Moreover in the Table 1.2 the maximin LHDs obtained by ESE approach are not reported in Table 1.2. As ESE approach performs relatively better compare to PD or SA, so we will compare ILS with ESE separately. It is observed that except dimension $k = 3$, in which PD performs better, ILS outperforms compared to other approaches considered. We observe that ILS is able to detect a very large amount of improved solutions with respect to the best-known ones [Currin C et al.]. This is, especially, true at large k values. For $k \geq 6$, with the exception of few numbers of *N values*, all the solutions returned by ILS are better compare to the best-known results. It is worthwhile to remark that for large (k, N) values the improvement of each LHD obtained by ILS approach is very significance.

Table 1.3: the comparison among several approaches for finding maximin LHDs for N=2 to 100 in each dimension k

	Number of best					Identical	Worse
k	***PD***	***SA***	***SA_ M***	***Web***	***ILS***	***ILS***	***ILS***
3	**61**	0	0	*65*	**14**	20	65
4	**02**	0	0	*47*	**34**	18	47
5	00	0	0	*11*	**78**	10	11
6	00	0		00	**90**	09	00
7	00	0		00	**92**	07	00
8		0		00	**93**	06	00
9		0		00	**93**	06	00
10		0		00	**92**	07	00

Though the performance of ILS approach is significantly better compare to other approaches consider here, but the approach will be effective if it is efficient i.e. the algorithm performs the job within acceptable time. So it is needed to comment about the computation times. It is worthwhile to mention here that there is no information regarding times to obtain the Web's results. Anyway, for this demand, the computational cost of the approaches is reported in the Table 1.4. It is noted that the elapsed time of ESE approach is not available. It is, however, quite clear that ILS is more computationally demanding with respect to PD and SA. Such higher costs are clearly rewarded in terms of quality of the results but the quality of the results might be wondered if the time restrictions are imposed on ILS. According to some further experiments that were performed, it would be realized that, especially at large k values, equivalent or better results with respect to the PD and SA ones, could quickly be reached by ILS [Jamali (2009)]. Therefore, it seems that at large k values even few and short runs of ILS are able to deliver results better than those reached by PD and SA. That is ILS approach outperforms compare to other approaches considered regarding L^2 distance measure.

Table 1.4: Comparison of computational cost

Total Elapsed Time (hrs)			
k	PD	SA	ILS
3	145	500	164
4	61	181	507
5	267	152	767
6	108	520	1235
7	232	246	698
8	--	460	846
9	--	470	1087
10	--	470	1166

Now we will compare the performance of ILS with respect to ESE regarding maximin LHDs by summering the above maximin LHDs values for ILS and ESE approach. Table
1.5 displays the intimate comparison between ILS and ESE approach regarding maximin LHD's values. It is observed that except dimension 4, in which performance of both approaches are comparable, ILS always outperforms significantly. Moreover we notice that for k > 5, the almost all maximin LHDs obtained by ILS approach are the better.

Table 1.5: Comparison between ESE and ILS regarding Maximin LHDS

N= ,…, 00	No. of Best LHDS in	
k	**ESE**	**ILS**
3	24	43
4	45	35
5	11	77
6	0	92
7	1	91
8	0	92
9	0	93
10	0	92

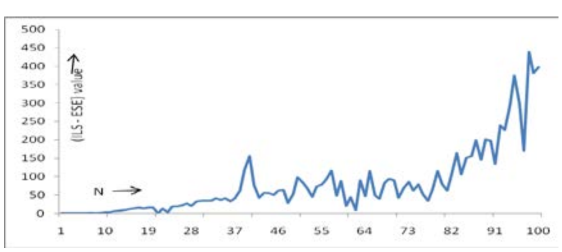

Figure 1.1 Effect of *N* on the performance of ILS approach upon ESE approach for $k = 10$

Now what are the effects of increasing of *N* on the performance of ILS approach upon ESE approach is depicted in the Figure 1.1. For this contest we consider k =10 and N= 2,…., 100. In the Figure 1.1 the horizontal line indicates N (number of design point) and vertical line indicates the difference between the D_1 value of MLH_ILS and MLH_ESE (MLH_ILS - MLH_ESE). It is observed in the figure that there is a significant effect on N. For the increasing of *N*, ILS approach find out much batter LHDs compare to ESE approach. From the above discussion it is clear that ILS approach is state-of arts regarding maximin optimality in Euclidian distance measure as well as computational cost.

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