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MULTI-FREQUENCY ACTIVE CONTROL OF RESONANCES PIEZOELECTRIC SANDWICH BEAMS

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ABSTRACT

In this paper, we study the primary resonances of a sandwich beam piezoelectric/elastic/piezoelectric subjected to two excitation frequencies. For this purpose, the feedback potential via the piezoelectric sensor and actuator is used. The dynamic model of the sandwich beam results in a differential equation not linear, the Galerkin approximation allows us to discretize this equation of movement. The multiple scale method is used to obtain solutions approached for strong excitation. Analytical frequency-amplitude relationships and phase-amplitude are given. The static and dynamic stability criteria are studied, the critical displacement and the associated excitation amplitude are given analytically. The effects of feedback parameters in this work are analyzed

1 INTRODUCTION

Adaptive structures with piezoelectric layers are widely used for vibration control and also for mechanical systems. vibration control and also for mechanical systems [1-3]. However, the work carried out on this type of structure with piezoelectric layers or corrective layers is based on linear theory. Thus, modelling the non linear behaviour when subjected to higher loads and large excitations is not accurate. excitations is not accurate. When vibration is introduced into a structure, it can reach a dangerous amplitude, so it is necessary to control and reduce these vibrations reduce these vibrations that are harmful to the structure. Many researchers have addressed the question in an attempt to model the dynamic behaviour of beams and plates with piezolaminate layers, Moita et al [2], who used the updated Lagrangian formulation combined with the Newton-Raphson technique. combined with the Newton-Raphson technique. Gao [3] and Shen proposed a technique based on the Lagrangian and the differential principle of virtual velocity. In these earlier earlier studies, finite element formulations were proposed for nonlinear transient transient vibrations of composite structures with piezoelectric materials. Active vibration control of piezoelectric/elastic/piezoelectric sandwich beams has been studied by S. Belouettar et al [1] using a simplified model for single-frequency excitation. Using structures with piezoelectric actuators and sensors and taking into account taking into account geometrical non-linearities, a model for the control of nonlinear vibration control model has been obtained using proportional feedback control derived from of the electrical potential. A mathematical methodology, based on the method of multiple scales for non-linear vibration control and stability studies have been developed as part of this work. developed as part of this work. A system of complex amplitude and phase equations taking account of geometric non-linearity parameters and the piezoelectric effect has been obtained.piezoelectric effect. Simultaneous resonance control has been developed. Feedback effects have been analysed for small and large vibration amplitudes of sandwich beams. Nowadays, a number of authors have devoted their work to the study of microstructure vibrations Younis and Nayfeh [4]; Farokhi et al. [5]; Askari [6]; Askari et al [7]; Farokhi and Ghayesh, [8 9]; Ghayesh and Farokhi, [9 10]; Han et al [11]; Ghorbanpour Arani et al [12]. The harmonic response for a microbeam was obtained by Younis and Nayfeh [13]. They studied the stability of the amplitude corresponding to the frequency response using the perturbation method. Various methods were used to solve the non-linear equation of the micro-madriers. Younesian et al [14 15] used an innovative innovative method to study the generalised form of the nonlinear oscillator. Xia et al [16] applied Hamilton's principle. The perturbation technique and the method of multiple scales were used to study the behaviour of microbridge electrostatic actuators [17]. In a recent study, Hassanpour et al investigated the free and forced nonlinear vibrations of a microbeam numerically and experimentally vibrations of a micro-beam numerically and experimentally [18 19]. In this based on the Euler-Bernoulli beam model, we propose to study the vibratory behaviour of a behaviour of a Duffing-Van der Poll type beam subjected to two excitation frequencies. In order to control the resonant amplitudes of vibration and thus in extending service life of laminated composite beams under periodic load or impact, the damping in the core layer play an important role. At the constituent level, the energy dissipation in fibre-reinforced composites is induced by different processes such as the viscoelastic behaviour of the matrix, the damping at the fibre–matrix interface, the damping due to damage, etc. At the laminate level, damping is depending on the constituent layer properties as well as the layer orientations, interlaminar effects, stacking sequence, etc Most of the studies of laminated composite beams are devoted to linear vibration and damping analysis. Earlier works on this subject are done by Gibson and Plunkett [1] and Gibson and Wilson [2]. A good overview on the available literature dealing with the vibration behaviour in presence of viscoelastic material can be found in the survey articles by Nakra [3,4]. In the earlier works, some of the important contributions are the works of Heng et al. [5], He and Rao [6], Rikards [7] and Bhimaraddi [8]. In all these works, a complex modulus, which consists of a real part representing elastic stiffness and an imaginary part representing dissipation, has been widely used to model the behaviour of linear viscoelastic materials under harmonic vibrations. With respect to the introduction of geometrical nonlinearity for beams with viscoelastic cores, Kovac et al. [9] and Hyer et al. [10,11] studied the nonlinear vibration of a damped sandwich beam. This study is based on a multi-mode Galerkin procedure coupled with the harmonic balance method. Sandwich and laminated composite beams have been analysed using the classical models developed for one-layer beams (solid beams)

These models are based on a theory that neglects transverse shear and normal strains and leads to the classical laminate theory (CLT) [12,13]. Due to the drawbacks of the CLT, a first order shear deformation theory (FSDT) has been proposed to take into account the transverse shear deformation [14–16]. The effects of the transverse shear deformation are pronounced for composite beams because of the high ratio of the extensional modulus to the transverse shearing modulus. The FSDT is widely used, and assumes a constant transverse shear strain in the thickness direction [17]. Therefore, a shear correction factor is generally used to adjust the transverse shear stiffness in dynamic analyses of laminates [18–21]. To avoid the use of a shear correction factor, higher order shear deformation theories (HSDTs) have been developed [22–24]. These theories are more realistic, since they give zero transverse shear stress condition at thetop and bottom surface boundaries of the structure. The HSDTs have been successfully and extensively applied to design of multi-layered structural components. The discontinuity of some mechanical properties in the thickness direction represents a flaw in these theories. Also, it should be emphasised that recent research [25,26] has shown soft-core sandwich plates. The HSDTs are therefore of limited values for analysing problems in which an accurate description of the transverse normal stress distribution and related consequences are of interest. To overcome such limitations, Kapuria et al. [27] have used zig-zag theories, satisfying the inter-laminar continuity of the transverse shear stresses, to predict the dynamic and buckling responses of laminated beams with arbitrary layouts. The aim of this work is to develop a simple consistent theory for the nonlinear vibration analysis of laminated composite beams with large amplitudes. This theory couples the harmonic balance technique to Galerkin procedure. The nonlinear geometrical effect due to axial forces caused by axial restraints is modelled using higher order zig-zag theories, which incorporate various shear function models for the shear deformation in the core.

2 MATHEMATICAL FORMULATION

$$\begin{cases} u^i(x,y,z,t) = u^i(x,y,t) + (z - z_i)\varphi_x \\ v^i(x,y,z,t) = 0 \\ w^i(x,y,z,t) = w^i(x,y,t) \end{cases} \tag{1}$$

With $i = e, A, S$ S = upper piezoelectric patch layer (Sensor)

e = elastic intermediate layer (Core)
A = lower piezoelectric patch layer (Actuator)
φ_x, φ_y = rotation of the normal of the median plane of the central layer
z_i = ordinate of the median axis of the with layer

$$\begin{cases} \varphi_x(x,y) = -\dfrac{\partial w}{\partial x} \\ \varphi_y(x,y) = -\dfrac{\partial w}{\partial y} \\ \varphi_z(x,y) = -\dfrac{\partial w}{\partial z} = 0 \end{cases} \quad \text{and} \quad z_S = -z_A = \frac{1}{2}(h_e + h_s) \tag{2}$$

$$\begin{cases} u_S(x,y,\dfrac{h_s}{2}) = u_e(x,y,\dfrac{h_s}{2}) \\ u_A(x,y,-\dfrac{h_s}{2}) = u_e(x,y,-\dfrac{h_s}{2}) \end{cases} \tag{3}$$

Hence the displacement fields in the layers

$$\begin{cases} U^i(x,y,z,t) = u_0{}^i(x,y,t) - (z - \dfrac{h_A + h_S}{2})\omega_{,x} \\ U^e(x,y,z,t) = u_0(x,y,t) + z\varphi(x,y,t) \\ w^i(x,y,z,t) = w_0(x,y,t) \end{cases} \tag{4}$$

2.2 DEFORMATION FIELD

Strain fields are derived from displacement fields. Considering uniaxial displacements, we have :

$$\begin{cases} \varepsilon_{1S}(x,y,t) = u_{0,x} + \dfrac{h_e}{2}\varphi_{,x} - \dfrac{h_S}{2}w_{,xx} - (z - z_S)w_{,xx} + \dfrac{1}{2}w^2{}_{,x} \\ \varepsilon_{1e}(x,y,t) = u_{0,x} + z\varphi_{,x} \\ \varepsilon_{1A}(x,y,t) = u_{0,x} - \dfrac{h_e}{2}\varphi_{,x} + \dfrac{h_S}{2}w_{,xx} - (z - z_A)w_{,xx} + \dfrac{1}{2}w^2{}_{,x} \end{cases} \tag{5}$$

2.3 PLANE STRESS STATES

At any point M in plane stress, the equations of Elasticity can be written as follows

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ 0 \\ 0 \\ 0 \\ \sigma_{xy} \end{Bmatrix} = \begin{bmatrix} C_{11}^E & C_{12}^E & C_{13}^E & 0 & 0 & C_{16}^E \\ C_{12}^E & C_{22}^E & C_{23}^E & 0 & 0 & C_{26}^E \\ C_{13}^E & C_{23}^E & C_{33}^E & 0 & 0 & C_{36}^E \\ 0 & 0 & 0 & C_{44}^E & C_{45}^E & 0 \\ 0 & 0 & 0 & C_{45}^E & C_{55}^E & 0 \\ C_{16}^E & C_{26}^E & C_{36}^E & 0 & 0 & C_{33}^E \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ 0 \\ 0 \\ \varepsilon_6 \end{Bmatrix} \tag{6}$$

$$\Rightarrow \begin{cases} \sigma_1 = C_{11}^E\varepsilon_1 + C_{12}^E\varepsilon_2 + C_{13}^E\varepsilon_3 + C_{16}^E\varepsilon_6 & (7.1) \\ \sigma_2 = C_{12}^E\varepsilon_1 + C_{22}^E\varepsilon_2 + C_{23}^E\varepsilon_3 + C_{26}^E\varepsilon_6 & (7.2) \\ 0 = C_{13}^E\varepsilon_1 + C_{23}^E\varepsilon_2 + C_{33}^E\varepsilon_3 + C_{36}^E\varepsilon_6 & (7.3) \\ \sigma_6 = C_{16}^E\varepsilon_1 + C_{26}^E\varepsilon_2 + C_{36}^E\varepsilon_3 + C_{66}^E\varepsilon_6 & (7.4) \end{cases}$$

Relation (7c) allows us to derive the parameter

$$\varepsilon_3 = -\left(\frac{C_{13}^E}{C_{33}^E}\varepsilon_1\right) + \left(\frac{C_{23}^E}{C_{33}^E}\varepsilon_2\right) + \left(\frac{C_{36}^E}{C_{33}^E}\varepsilon_3\right) \quad (8)$$

By replacing in (7a) , (7b), and (7d) we have:

$$\begin{cases} \sigma_1 = \left(C_{11}^E - \frac{\left(C_{13}^E\right)^2}{C_{33}^E}\right)\varepsilon_1 + \left(C_{12}^E - \frac{C_{13}^E C_{23}^E}{C_{33}^E}\right)\varepsilon_2 + \left(C_{16}^E - \frac{C_{13}^E C_{36}^E}{C_{33}^E}\right)\varepsilon_6 \\ \sigma_2 = \left(C_{12}^E - \frac{C_{23}^E C_{13}^E}{C_{33}^E}\right)\varepsilon_1 + \left(C_{22}^E - \frac{\left(C_{23}^E\right)^2}{C_{33}^E}\right)\varepsilon_2 + \left(C_{26}^E - \frac{C_{23}^E C_{36}^E}{C_{33}^E}\right)\varepsilon_6 \\ \sigma_6 = \left(C_{16}^E - \frac{C_{36}^E C_{13}^E}{C_{33}^E}\right)\varepsilon_1 + \left(C_{26}^E - \frac{C_{36}^E C_{23}^E}{C_{33}^E}\right)\varepsilon_2 + \left(C_{66}^E - \frac{\left(C_{36}^E\right)^2}{C_{33}^E}\right)\varepsilon_6 \end{cases}$$

Let's ask

$$C_{11}^* = \left(C_{11}^E - \frac{\left(C_{13}^E\right)^2}{C_{33}^E}\right) C_{22}^* = \left(C_{22}^E - \frac{\left(C_{23}^E\right)^2}{C_{33}^E}\right) C_{66}^* = \left(C_{66}^E - \frac{\left(C_{36}^E\right)^2}{C_{33}^E}\right) \quad (9)$$

$$C_{12}^* = C_{12}^E - \frac{C_{13}^E C_{23}^E}{C_{33}^E} \; C_{16}^* = C_{16}^E - \frac{C_{36}^E C_{13}^E}{C_{33}^E} \; C_{26}^* = C_{26}^E - \frac{C_{23}^E C_{36}^E}{C_{33}^E}$$

Equation (9) therefore becomes:

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_6 \end{Bmatrix} = \begin{bmatrix} C_{11}^* & C_{12}^* & C_{16}^* \\ C_{12}^* & C_{22}^* & C_{26}^* \\ C_{16}^* & C_{26}^* & C_{66}^* \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_6 \end{Bmatrix} \quad (10)$$

(Belouettar et al., 2008), (Benjeddou et al., 1997), (Azar et al., 2008), (Daya et al., 2005) in their work introduced the effect of geometric non-linearity by assuming moderate rotations. The non-linear relationship between strain and displacement is given by :

$\varepsilon = \varepsilon_0 - z(w_{,xx} + \frac{1}{2} w_{,xx} w_{,x}{}^2)$ and $\varepsilon_0 = u_{,x} + \frac{1}{2} w_{,x}{}^2$ (11)

The shear effect, which is carried by the axis (Oy), is neglected. The coupling effects between the mechanical and electrical properties are given by the following systems:

$$\begin{cases} \begin{pmatrix} \sigma_1 \\ D_3 \end{pmatrix} = \begin{bmatrix} c_{11}^* & -e_{31}^* \\ e_{31}^* & \in_{33}^* \end{bmatrix} \\ \in_{33}^* = \in_{33} + \frac{e_{33}^2}{c_{33}}; e_{31}^* = e_{31} - \frac{c_{13}}{c_{33}} e_{33}; c_{11}^* = c_{11} - \frac{c_{13}^2}{c_{33}} \end{cases} \quad (12)$$

$$\begin{cases} \sigma = c\varepsilon - e^t E \\ D = e\varepsilon + \in E \end{cases} \quad (13)$$

σ : Stress vector

ε : Linear deformations

$[e]$: Matrix of electrical constants or stresses

$[\in]^t$: Dielectric permittivity matrix for constant deformation

c : Elasticity matrix

According to Opinathan et al, 2000, the electric potential in the electric layer in the elastic layer is a function of the electric potentials at the surface and whether the circuit is open or closed. For a core with open-circuit electrodes, the potential is unknown.

$$D_3 = e^*_{31}\varepsilon + \in^*_{33} E_3 \quad (14)$$

$$D_3(z) \equiv 0 \Rightarrow e^*_{31}\varepsilon = -\in^*_{33} E_3$$

$$E_3 = -\frac{e^*_{31}}{\in_{33}}\varepsilon_0 u_{,x} + \frac{e^*_{31}}{\in_{33}} z(w_{,xx}\frac{1}{2}w_{,xx}w_{,x}{}^2) \quad (15)$$

By definition, the electric field derives from a potential $\vec{E} = -\overrightarrow{grad}\varphi$

$$\int_{z_-}^{z_+} d\varphi = -\int_{z_-}^{z_+} E dz \quad (16)$$

$$E_3(z) = -\frac{e^*_{31}}{\in_{33}}[\varepsilon_0 - z(w_{,xx} + \frac{1}{2}w_{,xx}w_{,x}{}^2)] \quad (17)$$

$$\left| \begin{array}{l} E(z_S) = -\frac{e^*_{31}}{\in_{33}}[u_{,x} + \frac{1}{2}w_{,x}{}^2(1 - z_S w_{,xx}) - z_S w_{,xx}] \\ E(z_A) = -\frac{e^*_{31}}{\in_{33}}[u_{,x} + \frac{1}{2}w_{,x}{}^2(1 - z_A w_{,xx}) - z_A w_{,xx}] \end{array} \right. \Rightarrow E(z_i) = -\frac{e^*_{31}}{\in_{33}}[u_{,x} + \frac{1}{2}w_{,x}{}^2(1 - z_i w_{,xx}) - z_i w_{,xx}]; i = A, S \quad (18)$$

$$\left| \begin{array}{l} E_3 = -\frac{\partial\varphi}{\partial z} \\ \Delta\varphi = \frac{e^*_{31}}{\in_{33}} h_i[u_{,x} + \frac{1}{2}w_{,x}{}^2(1 - z_i w_{,xx}) - z_i w_{,xx}] \end{array} \right. \Rightarrow \left| \begin{array}{l} E_3 = -\frac{e^*_{31}}{\in^*_{33}}[u_{,x} + \frac{1}{2}w_{,x}{}^2(1 - z w_{,xx}) - z w_{,xx}] \\ \frac{\Delta\varphi}{h_i} = \frac{e^*_{31}}{\in_{33}}[u_{,x} + \frac{1}{2}w_{,x}{}^2(1 - z_i w_{,xx}) - z_i w_{,xx}] \end{array} \right. \quad (19)$$

The core of our sandwich beam is assumed to be conductive with a uniform potential fixed at, zero. The potential in the transducer is given by the expression

$$\begin{array}{l} \varphi_S = \Delta\varphi = \frac{e^*_{31}}{\in^*_{33}} h_S \left[u_{,x} + \frac{1}{2}w_{,x}{}^2(1 - z_S w_{,xx}) - z_S w_{,xx} \right] \\ \Rightarrow \dot{\varphi}_S = \Delta\varphi = \frac{e^*_{31}}{\in^*_{33}} h_S \left[\dot{u}_{,x} + \dot{w}_{,x} w_{,x}(1 - z_S w_{,xx}) - \frac{1}{2}w_{,x}{}^2 z_S \dot{w}_{,xx} - z_S \dot{w}_{,xx} \right] \end{array} \quad (20)$$

The potential at the actuator is assumed to depend on the sensor potential by proportional feedback control according to the law :

$$\varphi_A = G_p\varphi_S + G_d\dot{\varphi}_S \quad (21)$$

Using equations (12) and (13), the electric fields in the sensor and actuator and following Azrar et al, (1993) are written as follows:

$$\begin{cases} E_3^S = -\frac{\varphi_S}{h_S} + \frac{e^*_{31}}{\in^*_{33}}(z - z_S)[w_{,xx} + \frac{1}{2}w_{,x}{}^2 w_{,xx}] \\ E_3^A = \frac{\varphi_A}{h_S} + \frac{e^*_{31}}{\in^*_{33}}(z - z_A)[w_{,xx} + \frac{1}{2}w_{,x}{}^2 w_{,xx}] \end{cases} \quad (22)$$

2.4 VARIATIONAL FORMULATION

The equations of a piezoelectric continuum can be derived from Hamilton's Principle where the Lagrangian is adapted to include electrical and mechanical contributions.

$$\delta\int_{t_0}^{t_1}\Pi dt=0\ ,\ \delta\Pi=\int_V \sigma\delta\varepsilon dV \tag{23}$$

$$\delta T=(\varphi S)_*\int_{t_1}^{t_2}\int_0^L[\dot{u}\delta u+\dot{w}\delta w]\,dxdt$$

$$\delta T=(\varphi S)_*\int_0^L[\dot{u}\delta u\Big|_{t_0}^{t_1}-\int_{t_0}^{t_1}\ddot{u}\delta udt]dx+(\varphi S)_*\int_0^L[\dot{w}\delta w\Big|_{t_0}^{t_1}-\int_{t_0}^{t_1}\ddot{w}\delta wdt]dx$$

$$\Rightarrow\delta T=(\varphi S)_*\int_0^L[\ddot{u}\delta u+\ddot{w}\delta w]dxdt \tag{24}$$

The equation (11) in (25) gives us :

$$W=\int_V\sigma_1\delta\varepsilon dV=\int_V\sigma_1\delta[\varepsilon_0-z(w_{,xx}+\frac{1}{2}w_{,xx}w_{,x}{}^2)]dV$$

$$W=\int_V\sigma_1\delta\varepsilon dV=\int_V\sigma_1\delta[\varepsilon_0-z(w_{,xx}+\frac{1}{2}w_{,xx}w_{,x}{}^2)]dV \tag{25}$$

$$W=\int_V\sigma_1\delta\varepsilon dV=W=\int_V\sigma_1\delta\varepsilon dV=\int_V\sigma_1\delta[\varepsilon_0-z(w_{,xx}+\frac{1}{2}w_{,xx}w_{,x}{}^2)]dV$$

$$=\int_{V_A}\sigma_1\delta[\varepsilon_0-z(w_{,xx}+\frac{1}{2}w_{,xx}w_{,x}{}^2)]dV_A+\int_{V_C}\sigma_1\delta[\varepsilon_0-z(w_{,xx}+\frac{1}{2}w_{,xx}w_{,x}{}^2)]dV_C+ \tag{26}$$

$$\int_{V_S}\sigma_1\delta[\varepsilon_0-z(w_{,xx}+\frac{1}{2}w_{,xx}w_{,x}{}^2)]dV_S$$

$$=\int_{V_A}\sigma_1\delta\varepsilon_0dV_A+\int_{V_S}\sigma_1\delta\varepsilon_0dV_S+\int_{V_C}\sigma_1\delta\varepsilon_0dV_C$$

$$-\int_{V_A}\sigma_1\delta z(w_{,xx}+\frac{1}{2}w_{,xx}w_{,x}{}^2)dV_A-\int_{V_S}\sigma_1\delta z(w_{,xx}+\frac{1}{2}w_{,xx}w_{,x}{}^2)dV_S-\int_{V_C}\sigma_1\delta z(w_{,xx}+\frac{1}{2}w_{,xx}w_{,x}{}^2)dV_C \tag{27}$$

We know that $dV_S=S_Sd_x$ and $dV_A=S_Ad_x$

$$N_1^S=\int_0^L\sigma_1S_S\delta\varepsilon_0dx \qquad M_1^S=-\int_{V_S}\sigma\,\delta z(w_{,xx}+\frac{1}{2}w_{,xx}w_{,x}{}^2)\,dV_S$$

$$N_1^A=\int_0^L\sigma_1S_A\delta\varepsilon_0dx \qquad M_1^A=-\int_{V_A}\sigma\,\delta z(w_{,xx}+\frac{1}{2}w_{,xx}w_{,x}{}^2)\,dV_A$$

$$N_1^C=\int_0^L\sigma_1S_C\delta\varepsilon_0dx \qquad M_1^C=-\int_{V_C}\sigma\,\delta z(w_{,xx}+\frac{1}{2}w_{,xx}w_{,x}{}^2)\,dV_C$$

Determination of Normal force N

$$N_1^S = \int_0^L \sigma_1 S_S \delta\varepsilon_0 dx = \int_0^L S_S (c_{11}^* \varepsilon - e_{31}^* E_3^S) \delta\varepsilon_0 dx$$
$$= \int_0^L S_S c_{11}^* \varepsilon \delta\varepsilon_0 dx - \int_0^L S_S e_{31}^* E_3^S \delta\varepsilon_0 dx$$
$$= \int_0^L S_S c_{11}^* (\varepsilon_0 - z(w_{,xx} + \frac{1}{2} w_{,xx} w_{,x}{}^2)) \delta\varepsilon_0 dx - \int_0^L S_S e_{31}^* (\frac{-\varphi_S}{h_S} + \frac{e_{31}^*}{\epsilon_{33}^*}(z - z_S)(w_{,xx} + \frac{1}{2} w_{,xx} w_{,x}{}^2)) \delta\varepsilon_0 dx \quad (28)$$
$$= \int_0^L S_S c_{11}^* \varepsilon_0 \delta\varepsilon_0 dx - \int_0^L S_S c_{11}^* z(w_{,xx} + \frac{1}{2} w_{,xx} w_{,x}{}^2) \delta\varepsilon_0 dx + \int_0^L S_S e_{31}^* \frac{\varphi_S}{h_S} \delta\varepsilon_0 dx - \int_0^L S_S \frac{e_{31}^{*\,2}}{\epsilon_{33}^*}(z - z_S)(w_{,xx} + \frac{1}{2} w_{,xx} w_{,x}{}^2) \delta\varepsilon_0 dx$$
$$= \int_0^L S_S c_{11}^* \varepsilon_0 \delta\varepsilon_0 dx - \int_0^L S_S c_{11}^* z w_{,xx} \delta\varepsilon_0 dx - \int_0^L S_S c_{11}^* \frac{z}{2} w_{,xx} w_{,x}{}^2 \delta\varepsilon_0 dx + \int_0^L S_S e_{31}^* \frac{\varphi_S}{h_S} \delta\varepsilon_0 dx -$$
$$\int_0^L S_S \frac{e_{31}^{*\,2}}{\epsilon_{33}^*} z w_{,xx} - \int_0^L S_S \frac{e_{31}^{*\,2}}{2\epsilon_{33}^*} z w_{,xx} w_{,x}{}^2 \delta\varepsilon_0 dx \delta\varepsilon_0 + \int_0^L S_S \frac{e_{31}^{*\,2}}{\epsilon_{33}^*} z_S w_{,xx} \delta\varepsilon_0 dx + \int_0^L S_S \frac{e_{31}^{*\,2}}{2\epsilon_{33}^*} z_S w_{,xx} w_{,x}{}^2 \delta\varepsilon_0 dx$$

$$\Rightarrow N_1^S = \int_0^L S_S c_{11}^* \varepsilon_0 \delta\varepsilon_0 dx - \int_0^L S_S c_{11}^* z w_{,xx} \delta\varepsilon_0 dx - \int_0^L S_S c_{11}^* \frac{z}{2} w_{,xx} w_{,x}{}^2 \delta\varepsilon_0 dx + \int_0^L S_S e_{31}^* \frac{\varphi_S}{h_S} \delta\varepsilon_0 dx$$
$$- \int_0^L S_S \frac{e_{31}^{*\,2}}{\epsilon_{33}^*} z w_{,xx} - \int_0^L S_S \frac{e_{31}^{*\,2}}{2\epsilon_{33}^*} z w_{,xx} w_{,x}{}^2 \delta\varepsilon_0 dx \delta\varepsilon_0 + \int_0^L S_S \frac{e_{31}^{*\,2}}{\epsilon_{33}^*} z_S w_{,xx} \delta\varepsilon_0 dx + \int_0^L S_S \frac{e_{31}^{*\,2}}{2\epsilon_{33}^*} z_S w_{,xx} w_{,x}{}^2 \delta\varepsilon_0 dx$$

By introducing the law of equation (21):

$$\Rightarrow N_1^A = \int_0^L S_A c_{11}^* \varepsilon_0 \delta\varepsilon_0 dx - \int_0^L S_A c_{11}^* z w_{,xx} \delta\varepsilon_0 dx - \int_0^L S_A c_{11}^* \frac{z}{2} w_{,xx} w_{,x}{}^2 \delta\varepsilon_0 dx - \int_0^L S_A e_{31}^* \frac{(G_p \varphi_S + G_p \dot{\varphi}_S)}{h_A} \delta\varepsilon_0 dx$$
$$- \int_0^L S_A \frac{e_{31}^{*\,2}}{\epsilon_{33}^*} z w_{,xx} \varepsilon_0 \delta\varepsilon_0 dx + \int_0^L S_A \frac{e_{31}^{*\,2}}{\epsilon_{33}^*} z^2 w_{,xx} \delta\varepsilon_0 dx + \int_0^L S_A \frac{e_{31}^{*\,2}}{\epsilon_{33}^*} \frac{z^2}{2} w_{,xx} w_{,x}{}^2 \delta\varepsilon_0 dx \quad (29)$$
$$+ \int_0^L S_A \frac{e_{31}^{*\,2}}{\epsilon_{33}^*} z_A w_{,xx} \varepsilon_0 \delta\varepsilon_0 dx - \int_0^L S_A \frac{e_{31}^{*\,2}}{\epsilon_{33}^*} z z_A w_{,xx} \varepsilon_0 \delta\varepsilon_0 dx - \int_0^L S_A \frac{e_{31}^{*\,2}}{2\epsilon_{33}^*} z z_A w_{,xx} w_{,x}{}^2 \delta\varepsilon_0 dx$$
$$= - \int_0^L S_A e_{31}^* \frac{G_p \varphi_S}{h_A} \delta\varepsilon_0 dx - \int_0^L S_A e_{31}^* \frac{G_p \dot{\varphi}_S}{h_A} \delta\varepsilon_0 dx + \int_0^L S_A c_{11}^* \varepsilon_0 \delta\varepsilon_0 dx - \int_0^L S_A c_{11}^* z w_{,xx} \delta\varepsilon_0 dx - \int_0^L S_A c_{11}^* \frac{z}{2} w_{,xx} w_{,x}{}^2 \delta\varepsilon_0 dx$$
$$- \int_0^L S_A \frac{e_{31}^{*\,2}}{\epsilon_{33}^*} z w_{,xx} \varepsilon_0 \delta\varepsilon_0 dx + \int_0^L S_A \frac{e_{31}^{*\,2}}{\epsilon_{33}^*} z^2 w_{,xx} \delta\varepsilon_0 dx + \int_0^L S_A \frac{e_{31}^{*\,2}}{\epsilon_{33}^*} \frac{z^2}{2} w_{,xx} w_{,x}{}^2 \delta\varepsilon_0 dx$$
$$+ \int_0^L S_A \frac{e_{31}^{*\,2}}{\epsilon_{33}^*} z_A w_{,xx} \varepsilon_0 \delta\varepsilon_0 dx - \int_0^L S_A \frac{e_{31}^{*\,2}}{\epsilon_{33}^*} z z_A w_{,xx} \varepsilon_0 \delta\varepsilon_0 dx - \int_0^L S_A \frac{e_{31}^{*\,2}}{2\epsilon_{33}^*} z z_A w_{,xx} w_{,x}{}^2 \delta\varepsilon_0 dx$$

$$N_1^C = \int_0^L \sigma_1 S_C \delta\varepsilon_0 dx = \int_0^L S_C E_C \delta\varepsilon_0 dx \quad (30)$$

$$N_1 = N_1^S + N_1^A + N_1^C$$

$$N_1 = N_1^S + N_1^A + N_1^C = \int_0^L S_S c_{11}^* \varepsilon_0 \delta\varepsilon_0 dx - \int_0^L S_S c_{11}^* z w_{,xx} \delta\varepsilon_0 dx + \int_0^L S_S \frac{e_{31}^{*\,2}}{\epsilon_{33}^*} \varepsilon_0 \delta\varepsilon_0 dx - \int_0^L S_S \frac{e_{31}^{*\,2}}{\epsilon_{33}^*} z w_{,xx} \delta\varepsilon_0 dx$$
$$+ \int_0^L S_A c_{11}^* \varepsilon_0 \delta\varepsilon_0 dx - \int_0^L S_A c_{11}^* z w_{,xx} \delta\varepsilon_0 dx - \int_0^L S_A \frac{e_{31}^{*\,2}}{\epsilon_{33}^*} G_p \varepsilon_0 \delta\varepsilon_0 dx + \int_0^L S_A \frac{e_{31}^{*\,2}}{\epsilon_{33}^*} G_p z_S \delta\varepsilon_0 dx - \int_0^L S_A \frac{e_{31}^{*\,2}}{\epsilon_{33}^*} G_d \dot{\varepsilon}_0 \delta\varepsilon_0 dx \quad (31)$$
$$+ \int_0^L S_A G_d \frac{e_{31}^{*\,2}}{\epsilon_{33}^*} z_S \dot{w}_{,xx} \delta\varepsilon_0 dx + \int_0^L S_A \frac{e_{31}^{*\,2}}{\epsilon_{33}^*} z_A w_{,xx} \delta\varepsilon_0 dx - \int_0^L S_A \frac{e_{31}^{*\,2}}{\epsilon_{33}^*} z w_{,xx} \delta\varepsilon_0 dx + \int_0^L S_C E_C \delta\varepsilon_0 dx$$
$$= \left[\int_0^L S_C E_C \delta\varepsilon_0 dx + \int_0^L S_S c_{11}^* \delta\varepsilon_0 dx + \int_0^L S_S \frac{e_{31}^{*\,2}}{\epsilon_{33}^*} \left(1 - G_p\right) \delta\varepsilon_0 dx \right] \varepsilon_0 - \left[\int_0^L S_S \frac{e_{31}^{*\,2}}{\epsilon_{33}^*} \left(1 - G_p\right) z_S \delta\varepsilon_0 dx \right] w_{,xx}$$
$$- \int_0^L S_S \frac{e_{31}^{*\,2}}{\epsilon_{33}^*} G_d (\dot{\varepsilon}_0 - \dot{w}_{,xx}) \delta\varepsilon_0 dx$$

$$(ES)_{pe} = S_S \frac{e_{31}^{*\,2}}{\epsilon_{33}^*}, \; B_N = (ES)_{pe}(1 - G_p) z_S, \; (ES)_* = E_C S_C + 2c_{11}^* S_S + (ES)_{pe}(1 - G_p) \quad (32)$$

$$N=(ES)_{*}\varepsilon_0-B_N w_{,xx}-(ES)_{pe}G_d(\dot{u}_{,x}+\dot{w}_{,x}w_{,x}-\dot{w}_{,xx}z_S)$$

Determination of the moment M

The moment will be determined in the viscoelastic core and the two layers

$$M_C=-\int_{V_C} z[E_C\varepsilon]\delta(w_{,xx}+\frac{1}{2}w_{,xx}w_{,x}{}^2)dV=-\int_0^L\int_0^b\int_{-\frac{h_C}{2}}^{\frac{h_C}{2}} z[E_C\varepsilon]\delta w_{,xx}dxdydz$$

$$-\frac{1}{2}\int_0^L\int_0^b\int_{-\frac{h_C}{2}}^{\frac{h_C}{2}} z[E_C\varepsilon]\delta w_{,xx}w_{,x}{}^2dxdydz$$

$$=-\int_0^L bE_C\int_{-\frac{h_C}{2}}^{\frac{h_C}{2}} z(\varepsilon_0-zw_{,xx})\delta w_{,xx}dzdx-\frac{1}{2}\int_0^L bE_C\int_{-\frac{h_C}{2}}^{\frac{h_C}{2}} z(\varepsilon_0-zw_{,xx})\delta w_{,xx}w_{,x}{}^2dzdx$$

$$=-\int_0^L bE_C\int_{-\frac{h_C}{2}}^{\frac{h_C}{2}} z\varepsilon_0\delta w_{,xx}dzdx+\int_0^L bE_C\int_{-\frac{h_C}{2}}^{\frac{h_C}{2}} z^2 w_{,xx}\delta w_{,xx}dzdx-\frac{1}{2}\int_0^L bE_C\int_{-\frac{h_C}{2}}^{\frac{h_C}{2}} z\varepsilon_0\delta w_{,xx}w_{,x}{}^2dzdx$$

$$+\frac{1}{2}\int_0^L bE_C\int_{-\frac{h_C}{2}}^{\frac{h_C}{2}} z^2 w_{,xx}\delta w_{,xx}w_{,x}{}^2dzdx$$

$$\Rightarrow M_C=-\int_0^L bE_C\varepsilon_0[\frac{z^2}{2}]_{-\frac{h_C}{2}}^{\frac{h_C}{2}}\delta w_{,xx}dx+\int_0^L bE_C\varepsilon_0[\frac{z^3}{3}]_{-\frac{h_C}{2}}^{\frac{h_C}{2}}w_{,xx}\delta w_{,xx}dx \tag{33}$$

$$-\frac{1}{2}\int_0^L bE_C[\frac{z^2}{2}]_{-\frac{h_C}{2}}^{\frac{h_C}{2}}\delta w_{,xx}w_{,x}{}^2dx+\frac{1}{2}\int_0^L bE_C[\frac{z^3}{3}]_{-\frac{h_C}{2}}^{\frac{h_C}{2}}w_{,xx}\delta w_{,xx}w_{,x}{}^2dx$$

$$=\int_0^L bE_C\varepsilon_0\frac{h_C^3}{12}w_{,xx}\delta w_{,xx}dx+\frac{1}{2}\int_0^L bE_C\frac{h_C^3}{12}w_{,xx}\delta w_{,xx}w_{,x}{}^2dx$$

$$I_C=\frac{bh_C^3}{12}\qquad M_C=\int_0^L E_C I_C w_{,xx}\delta w_{,xx}dx+\frac{1}{2}\int_0^L bE_C I_C w_{,xx}\delta w_{,xx}w_{,x}{}^2dx \tag{34}$$

- Moment in the upper layer M^S

$$M_S=-\int_{V_S} z\sigma_1{}^S\delta(w_{,xx}+\frac{1}{2}w_{,xx}w_{,x}{}^2)dV_S=-\int_{V_S} z(c_{11}{}^*\varepsilon-e_{31}{}^*E_3{}^S)\delta(w_{,xx}+\frac{1}{2}w_{,xx}w_{,x}{}^2)dV_S= \tag{35}$$

$$-\int_{V_S} zc_{11}{}^*\varepsilon\,\delta w_{,xx}dV_S+\int_{V_S} ze_{31}{}^*E_3{}^S\delta w_{,xx}dV_S-\frac{1}{2}\int_{V_S} zc_{11}{}^*\varepsilon\, w_{,xx}w_{,x}{}^2dV_S+\frac{1}{2}\int_{V_S} ze_{31}{}^*E_3{}^S\delta w_{,xx}w_{,x}{}^2dV_S$$

Let's ask $A=-\int_{V_S} zc_{11}{}^*\varepsilon\delta w_{,xx}dV_S$ $B=+\int_{V_S} ze_{31}{}^*E_3{}^S\delta w_{,xx}dV_S$ $C=-\frac{1}{2}\int_{V_S} zc_{11}{}^*\varepsilon\,\delta w_{,xx}w_{,x}{}^2dV_S$ $D=+\frac{1}{2}\int_{V_S} ze_{31}{}^*E_3{}^S\delta w_{,xx}w_{,x}{}^2dV_S$

$$M_S=A+B+C+D=-\int_0^L c_{11}{}^*\varepsilon_0 S_S z_S\delta w_{,xx}dx+\int_0^L c_{11}{}^*I_S w_{,xx}\delta w_{,xx}dx+\int_0^L z_S{}^2S_S c_{11}{}^*w_{,xx}\delta w_{,xx}dx$$

$$-\int_0^L\varepsilon_0\frac{e_{31}^{*\,2}}{\in_{33}^*}S_S z_S\delta w_{,xx}dx+\int_0^L z_S{}^2\frac{e_{31}^{*\,2}}{\in_{33}^*}\delta w_{,xx}dx-\int_0^L\frac{e_{31}^{*\,2}}{\in_{33}^*}I_S\delta w_{,xx}dx-\int_0^L z_S S_S\frac{e_{31}^{*\,2}}{\in_{33}^*}w_{,xx}\delta w_{,xx}dx$$

$$-\int_0^L\varepsilon_0 c_{11}{}^*\frac{b}{4}h_s(h_c+h_s)w_{,xx}\,\delta w_{,xx}w_{,x}{}^2dx+\int_0^L c_{11}{}^*\frac{b}{6}(h_s^3+\frac{3}{4}h_c^2h_s+\frac{3}{2}h_ch_s^2)w_{,xx}\,\delta w_{,xx}w_{,x}{}^2dx \tag{36}$$

$$+\int_0^L z_S c_{11}{}^*\frac{b}{4}h_s(h_c+h_s)w_{,xx}\,\delta w_{,xx}w_{,x}{}^2dx-\frac{1}{4}\int_0^L\frac{e_{31}^{*\,2}}{\in_{33}^*}\varepsilon_0\,b\,h_s(h_c+h_s)\delta w_{,xx}w_{,x}{}^2dx$$

$$+\frac{1}{6}\int_0^L\frac{e_{31}^{*\,2}}{\in_{33}^*}b\,(h_s^3+\frac{3}{4}h_c^2h_s+\frac{3}{2}h_ch_s^2)\delta w_{,xx}w_{,x}{}^2dx$$

$z_S = (h_C + h_S)/2$ therefore $S_S = bh_S$

$$M_A = -\int_{V_A} z\sigma_1{}^A \delta(w_{,xx} + \frac{1}{2} w_{,xx} w_{,x}{}^2) dV_A = -\int_{V_A} z(c_{11}^* \varepsilon - e_{31}^* E_3^A)\delta(w_{,xx} + \frac{1}{2} w_{,xx} w_{,x}{}^2) dV_A \quad (37)$$

$$= -\int_{V_A} zc_{11}^* \, \varepsilon \, \delta w_{,xx} dV_A + \int_{V_A} z \, e_{31}^* E_3^A \delta w_{,xx} dV_A - \frac{1}{2}\int_{V_A} zc_{11}^* \varepsilon \delta w_{,xx} w_{,x}{}^2 dV_A + \frac{1}{2}\int_{V_A} ze_{31}^* E_3^A \delta w_{,xx} w_{,x}{}^2 dV_A$$

$$M_A = A' + B' + C' + D' = -\int_0^L c_{11}^* \varepsilon_0 S_A z_A \delta w_{,xx} dx + \int_0^L c_{11}{}^* I_S w_{,xx} \delta w_{,xx} dx + \int_0^L z_A{}^2 S_A c_{11}^* w_{,xx} \delta w_{,xx} dx$$

$$+\int_0^L \frac{e_{31}^{*\,2}}{\epsilon_{33}^*} G_p \varepsilon_0 z_A S_S \delta w_{,xx} dx - \int_0^L \frac{e_{31}^{*\,2}}{\epsilon_{33}^*} G_p z_S z_A S_S \delta w_{,xx} dx + \int_0^L \frac{e_{31}^{*\,2}}{\epsilon_{33}^*} I_S w_{,xx} \delta w_{,xx} dx$$

$$-\int_0^L \frac{e_{31}^{*\,2}}{\epsilon_{33}^*} z_A z_A S_S w_{,xx} \delta w_{,xx} dx + \int_0^L \frac{e_{31}^{*\,2}}{\epsilon_{33}^*} G_d \dot{\varphi}_s z_A S_S \delta w_{,xx} dx - \frac{1}{4}\int_0^L c_{11}^* \, \varepsilon_0 b(-h_A(\frac{h_C + h_S}{2})) w_{,xx} \delta w_{,xx} w_{,x}{}^2 dx$$

$$+\frac{1}{6}\int_0^L c_{11}^* \, \varepsilon_0 b(h_S{}^3 + \frac{3}{4} h_C{}^2 h_S + \frac{3}{2} h_S{}^2 h_C) w_{,xx} \delta w_{,xx} w_{,x}{}^2 dx + \frac{1}{4}\int_0^L c_{11}^* \, \varepsilon_0 b(-h_A(\frac{h_C + h_S}{2})) w_{,xx} \delta w_{,xx} w_{,x}{}^2 dx$$

$$+\frac{1}{2}\int_0^L \frac{e_{31}^{*\,2}}{\epsilon_{33}^*} G_p \varepsilon_0 z_A s_S \delta w_{,xx} w_{,x}{}^2 dx - \frac{1}{2}\int_0^L \frac{e_{31}^{*\,2}}{\epsilon_{33}^*} G_p z_A z_S s_S \delta w_{,xx} w_{,x}{}^2 dx + \frac{1}{2}\int_0^L \frac{e_{31}^{*\,2}}{\epsilon_{33}^*} I_S w_{,xx} \delta w_{,xx} w_{,x}{}^2 dx$$

$$-\frac{1}{2}\int_0^L \frac{e_{31}^{*\,2}}{\epsilon_{33}^*} z_A z_A S_S w_{,xx} \delta w_{,xx} w_{,x}{}^2 dx + \frac{1}{2}\int_0^L \frac{e_{31}^{*\,2}}{\epsilon_{33}^*} G_d \dot{\varphi}_s z_A S_S w_{,xx} \delta w_{,xx} w_{,x}{}^2 dx$$

$$\Rightarrow M = M_C + M_S + M_A$$

$$= \int_0^L bE_C \varepsilon_0 \frac{h_C^3}{12} w_{,xx} \delta w_{,xx} dx + \frac{1}{2}\int_0^L bE_C \frac{h_C^3}{12} w_{,xx} \delta w_{,xx} w_{,x}{}^2 dx - \int_0^L c_{11}{}^* \varepsilon_0 S_S z_S \delta w_{,xx} dx + \int_0^L c_{11}{}^* I_S w_{,xx} \delta w_{,xx} dx$$

$$+\int_0^L z_S{}^2 S_S c_{11}{}^* w_{,xx} \delta w_{,xx} dx - \int_0^L \varepsilon_0 \frac{e_{31}^{*\,2}}{\epsilon_{33}^*} S_S z_S \delta w_{,xx} dx + \int_0^L z_S{}^2 \frac{e_{31}^{*\,2}}{\epsilon_{33}^*} \delta w_{,xx} dx - \int_0^L \frac{e_{31}^{*\,2}}{\epsilon_{33}^*} I_S \delta w_{,xx} dx$$

$$-\int_0^L z_S S_S \frac{e_{31}^{*\,2}}{\epsilon_{33}^*} w_{,xx} \delta w_{,xx} dx - \int_0^L \varepsilon_0 c_{11}{}^* \frac{b}{4} h_s (h_c + h_s) w_{,xx} \, \delta w_{,xx} w_{,x}{}^2 dx$$

$$+\int_0^L c_{11}{}^* \frac{b}{6}(h_s^3 + \frac{3}{4} h_c^2 h_s + \frac{3}{2} h_c h_s^2) w_{,xx} \, \delta w_{,xx} w_{,x}{}^2 dx + \int_0^L z_S c_{11}{}^* \frac{b}{4} h_s (h_c + h_s) w_{,xx} \, \delta w_{,xx} w_{,x}{}^2 dx$$

$$-\frac{1}{4}\int_0^L \frac{e_{31}^{*\,2}}{\epsilon_{33}^*} \varepsilon_0 \, b \, h_s (h_c + h_s) \delta w_{,xx} w_{,x}{}^2 dx + \frac{1}{6}\int_0^L \frac{e_{31}^{*\,2}}{\epsilon_{33}^*} b \, (h_s^3 + \frac{3}{4} h_c^2 h_s + \frac{3}{2} h_c h_s^2) \delta w_{,xx} w_{,x}{}^2 dx$$

$$-\int_0^L c_{11}^* \varepsilon_0 S_A z_A \delta w_{,xx} dx + \int_0^L c_{11}{}^* I_S w_{,xx} \delta w_{,xx} dx + \int_0^L z_A{}^2 S_A c_{11}^* w_{,xx} \delta w_{,xx} dx$$

$$+\int_0^L \frac{e_{31}^{*\,2}}{\epsilon_{33}^*} G_p \varepsilon_0 z_A S_S \delta w_{,xx} dx - \int_0^L \frac{e_{31}^{*\,2}}{\epsilon_{33}^*} G_p z_S z_A S_S \delta w_{,xx} dx + \int_0^L \frac{e_{31}^{*\,2}}{\epsilon_{33}^*} I_S w_{,xx} \delta w_{,xx} dx$$

$$-\int_0^L \frac{e_{31}^{*\,2}}{\epsilon_{33}^*} z_A z_A S_S w_{,xx} \delta w_{,xx} dx + \int_0^L \frac{e_{31}^{*\,2}}{\epsilon_{33}^*} G_d \dot{\varphi}_s z_A S_S \delta w_{,xx} dx - \frac{1}{4}\int_0^L c_{11}^* \, \varepsilon_0 b(-h_A(\frac{h_C + h_S}{2})) w_{,xx} \delta w_{,xx} w_{,x}{}^2 dx$$

$$+\frac{1}{6}\int_0^L c_{11}^{*}\,\varepsilon_0 b(h_S{}^3+\frac{3}{4}h_C{}^2h_S+\frac{3}{2}h_S{}^2h_C)w_{,xx}\delta w_{,xx}w_{,x}{}^2dx+\frac{1}{4}\int_0^L c_{11}^{*}\,\varepsilon_0 b(-h_A(\frac{h_C+h_S}{2}))w_{,xx}\delta w_{,xx}w_{,x}{}^2dx$$

$$+\frac{1}{2}\int_0^L \frac{e_{31}^{*\,2}}{\epsilon_{33}^{*}}G_p\varepsilon_0 z_A s_S\delta w_{,xx}w_{,x}{}^2dx-\frac{1}{2}\int_0^L \frac{e_{31}^{*\,2}}{\epsilon_{33}^{*}}G_p z_A z_S s_S\delta w_{,xx}w_{,x}{}^2dx+\frac{1}{2}\int_0^L \frac{e_{31}^{*\,2}}{\epsilon_{33}^{*}}I_S w_{,xx}\delta w_{,xx}w_{,x}{}^2dx \quad (38)$$

$$-\frac{1}{2}\int_0^L \frac{e_{31}^{*\,2}}{\epsilon_{33}^{*}}z_A z_A S_S w_{,xx}\delta w_{,xx}w_{,x}{}^2dx+\frac{1}{2}\int_0^L \frac{e_{31}^{*\,2}}{\epsilon_{33}^{*}}G_d\dot{\varphi}_s z_A S_S w_{,xx}\delta w_{,xx}w_{,x}{}^2dx$$

By integrating considerations $z_A=-z_S\ \ h_S=h_A\ \ S_A=S_S$

$$M=-\left[\int_0^L S_S\,\frac{e_{31}^{*\,2}}{\epsilon_{33}^{*}}z_S\left(1+G_p\right)dx\right]\varepsilon_0-\int_0^L\frac{e_{31}^{*\,2}}{\epsilon_{33}^{*}}G_d\left(\dot{u}_{,x}+w_{,x}\dot{w}_{,x}-z_S\dot{w}_{,xx}\right)z_S S_S\delta w_{,xx}dx \quad (39)$$

$$+\left[\int_0^L E_C I_C\delta w_{,xx}dx+2c_{11}^{*}I_S\int_0^L\delta w_{,xx}dx+2c_{11}^{*}S_S z_S^2+\frac{e_{31}^{*\,2}}{\epsilon_{33}^{*}}\left(2I_S+(G_p+1)z_S^2S_S\right)\right]w_{,xx}$$

$$\left(ES\right)_{pe}=\frac{e_{31}^{*\,2}}{\epsilon_{33}^{*}} \quad (40)$$

$$B_M=\left(ES\right)_{pe}\left(1+G_p\right)z_S$$

$$\left(EI\right)_{*}=E_C I_C+2c_{11}^{*}\left(I_S+S_S z_S^2\right)+\frac{\left(ES\right)_{pe}}{S_S}\left(2I_S+(G_p+1)z_S^2S_S\right) \quad (41)$$

we obten $M=-B_M\varepsilon_0-\left(ES\right)_{pe}G_d\left(\dot{u}_{,x}+w_{,x}\dot{w}_{,x}-z_S\dot{w}_{,xx}\right)z_S+\left(EI\right)_{*}w_{,xx}$

$$\int_0^L N\delta\varepsilon_0 dx+\int_0^L M\delta w_{,xx}dx=\int_0^L F_X\delta u dx+\int_0^L F_Z\delta u dx-\rho S\int_0^L(\ddot{u}\delta u+\ddot{w}\delta w)dx \quad (42)$$

$$H=T-U+Wf_{ext} \quad (43)$$

$$T=\frac{1}{2}\int_0^L V^2dm=\frac{1}{2}\int_0^L\rho(\dot{u}^2+\dot{w}^2)dV=\frac{1}{2}\int_0^L(\rho S)_*(\dot{u}^2+\dot{w}^2)dx$$

$$\int_{t_0}^{t_1}\delta T dt=\frac{1}{2}\int_{t_0}^{t_1}\int_0^L(\rho S)_*\delta(\dot{u}^2+\dot{w}^2)dx=-\int_{t_0}^{t_1}\int_0^L(\rho S)_*\delta[\ddot{u}\delta u+\ddot{w}\delta w]dx$$

$$\delta\varepsilon_0=\delta u_{,x}+w_{,x}\delta w_{,x}$$

$$\int_0^L N\delta\varepsilon_0 dx=\int_0^L(N\delta u_{,x}+Nw_{,x}\delta w_{,x})dx \quad (44)$$

$$\int_0^L N\delta u_{,x}dx=[N\delta u]_0^L-\int_0^L N_{,x}\delta u dx$$

$$\int_0^L N\delta w_{,x}\delta w_{,x}dx=[Nw_{,x}\delta w]_0^L-\int_0^L[N_{,x}w_{,x}+Nw_{,xx}]\delta w dx=[Nw_{,x}\delta w]_0^L-\int_0^L(Nw_{,x})_{,x}\delta w dx$$

$$\begin{cases}N=\frac{1}{2}(ES)_*w_{,x}{}^2-B_N w_{,xx}-(ES)_{pe}G_d(w_{,x}\dot{w}_{,x}-\ddot{w}_{,xx}z_s)\\ M=-\frac{1}{2}B_M w_{,x}{}^2+(EI)_*w_{,xx}-(ES)_{pe}G_d z_s(w_{,x}\dot{w}_{,x}-\ddot{w}_{,xx}z_s)\end{cases} \quad (45)$$

$$N(t)=\int_0^L \frac{1}{2}(ES)_* w_{,x}{}^2 - B_N w_{,xx} - (ES)_{pe} G_d (w_{,x}\dot{w}_{,x} - \ddot{w}_{,xx} z_s) dx$$
$$=\int_0^L \frac{1}{2}(ES)_* w_{,x}{}^2 dx - \int_0^L B_N w_{,xx} dx - (ES)_{pe} \int_0^L G_d (w_{,x}\dot{w}_{,x} - \ddot{w}_{,xx} z_s) dx \quad (46)$$
$$=\frac{1}{2L}(ES)_* \int_0^L w_{,x}{}^2 dx - \frac{B_N}{L}\int_0^L w_{,xx} dx - \frac{(ES)_{pe}}{L} G_d \int_0^L G_d (w_{,x}\dot{w}_{,x} - \ddot{w}_{,xx} z_s) dx$$

$$M_{,xx} - N(t) w_{,xx} = F_z - (\rho S)\ddot{w}$$

$$-B_M w_{,xx}{}^2 - B_M w_{,x} w_{,xxx} + (EI) w_{,xxxx} - (ES)_{pe} G_d z_s (w_{,xxx}\dot{w}_{,x} + w_{,xx}\dot{w}_{,xx} + w_{,xx}\dot{w}_{,xx} + w_{,x}\dot{w}_{,xxx} - \dot{w}_{,xxxx} z_s)$$
$$-N(t) w_{,xx} = F_z - (\rho S)\ddot{w}$$

$$(\rho S)\ddot{w} + (EI)_* w_{,xxxx} - B_M (w_{,xx}{}^2 + w_{,x} w_{,xxx}) - N(t) w_{,xx}$$
$$-(ES)_{pe} G_d z_s (w_{,xxx}\dot{w}_{,x} + w_{,xx}\dot{w}_{,xx} + w_{,xx}\dot{w}_{,xx} + w_{,x}\dot{w}_{,xxx} - \dot{w}_{,xxxx} z_s) = F_z \quad (47)$$

$$\ddot{q}(t) + 2\mu\dot{q}(t) + \omega_1^2 q(t) + \alpha_2 q^2(t) + \alpha_3 q^3(t) + \alpha_4 q(t)\dot{q}(t) + \alpha_5 q^2(t)\dot{q}(t) = \sum_{i=1}^{2} F_i \cos(\omega_i t + \varphi_i) \quad (48)$$

2.5 MULTI-SCALE METHOD.

We will study the effects of gain parameters on frequency, phase, time response and instability. To this end, an approximate solution of equation (5) will be presented at different time scales using the multi-scale method. An analysis of the primary and secondary resonance will be carried out in the case of an elastic piezoelectric sandwich beam subjected to two excitation frequencies

Primary resonance where $F_1 = F_2$ and $\gamma_1 = \gamma_2$

$$\ddot{q}(t) - \omega_1^2 q(t) = -\varepsilon \begin{bmatrix} 2\mu\dot{q}(t) + \alpha_2 q^2(t) + \alpha_3 q^3(t) + \alpha_4 q(t)\dot{q}(t) + \\ \alpha_5 q^2(t)\dot{q}(t) + F_1\cos(\omega_1 t + \varphi_1) + F_2\cos(\omega_2 t + \varphi_2) \end{bmatrix} \quad (49)$$

This equation has a general first-order solution in the form: of time is defined as follows:

$$\begin{cases} T_n = \varepsilon^n t = \varepsilon^0 t + \varepsilon^1 t + \varepsilon^2 t + \ldots\ldots\ldots\ldots = T_0 + T_1 + T_2 + \ldots\ldots\ldots \\ T_0 = \varepsilon^0 t;\ T_1 = \varepsilon^1 t \end{cases} \quad (50)$$

With: $n \in N$ Note that is a small dimensionless parameter. The first approximation to the solution of the equation is sought in the form of an expansion in powers of ε :

$$q(t,\varepsilon) = \varepsilon^0 q_0(T_0, T_{1,}\ldots\ldots) + \varepsilon^1 q_1(T_0, T_{1,}\ldots\ldots) + \varepsilon^2 q_1(T_0, T_{1,}\ldots\ldots) + \ldots\ldots\ldots \quad (51)$$

For fast timescales $T_0 = t$, and for slow timescales, $T_1 = \varepsilon t$, will be introduced and the derivative operations can be written as follows: The time derivatives are written as :

$$\begin{cases} \dfrac{d}{dT} = \varepsilon^0 \dfrac{\partial}{T_0} + \varepsilon^1 \dfrac{\partial}{T_1} + \varepsilon^2 t + \ldots\ldots\ldots\ldots \\ \dfrac{d^2}{dT_n{}^2} = \varepsilon^0 D_0 + 2\varepsilon D_0 D_1 + \ldots\ldots\ldots \end{cases} \quad (52)$$

$$\begin{cases} \dfrac{d}{dT} = \varepsilon^0 D_0 + \varepsilon^1 D_1 + \varepsilon^2 D_2 + ... \\ \dfrac{d^2}{dT_n^{\ 2}} = \varepsilon^0 D_0 + 2\varepsilon^1 D_0 D_1 + \varepsilon^2 (2D_2 D_0 + D_1^2) + ... \end{cases} \tag{53}$$

$$\begin{cases} D_0^2 q_0 + \omega_1^2 q_0 = 0 & (54.1) \\ D_0 q_1 + \omega_1^2 q_1 - 2D_1 D_0 q_1 - 2\mu D_0 q_1 - \alpha_2 q_1^2 - \alpha_3 q_1^2 & \\ -\alpha_4 q_1 D_0 q_1 + \alpha_5 q_1^5 D_0 q_1 + F_1 \cos(\omega_1 t - \varphi_1) + F_2 \cos(\omega_2 t - \varphi_2) = 0 & (54.2) \end{cases}$$

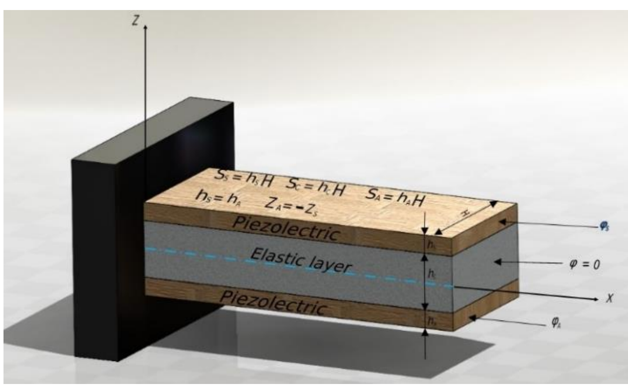
$$\begin{aligned} D_0^2 q_1 + \omega_l^2 q_1 &= -(2i\omega_l (A' + \mu A)) + (3A\alpha_3 A\bar{A} + 2iA\alpha_5 \omega_l A\bar{A}) e^{i\omega_l T_0} \\ &+ (\alpha_3 + i\omega_l \alpha_5) A^3 e^{3i\omega_l T_0} + (\alpha_2 + i\omega_l \alpha_4) A e^{3i\omega_l T_0} + (2\alpha_2 + i\omega_l \alpha_4) A\bar{A} + \frac{F_1}{2} e^{i\omega_1 T_0} + \frac{F_2}{2} e^{i\omega_2 T_0} \end{aligned} \tag{55}$$

$$\begin{cases} \dfrac{d}{dT} = \varepsilon^0 \dfrac{\partial}{T_0} + \varepsilon^1 \dfrac{\partial}{T_1} + \varepsilon^2 t + ... \\ \dfrac{d^2}{dT_n^{\ 2}} = \varepsilon^0 D_0 + 2\varepsilon D_0 D_1 + ... \end{cases} \tag{56}$$

$$\begin{aligned} D_0^2 q_1 + \omega_l^2 q_1 &= -(2i\omega_l (A' + \mu A)) + (3A\alpha_3 A\bar{A} + 2iA\alpha_5 \omega_l A\bar{A}) e^{i\omega_l T_0} \\ &+ (\alpha_3 + i\omega_l \alpha_5) A^3 e^{3i\omega_l T_0} + (\alpha_2 + i\omega_l \alpha_4) A e^{3i\omega_l T_0} + (2\alpha_2 + i\omega_l \alpha_4) A\bar{A} + \frac{F_1}{2} e^{i\omega_1 T_0} + \frac{F_2}{2} e^{i\omega_2 T_0} \end{aligned} \tag{57}$$

$$q(t,\varepsilon) = \varepsilon^0 q_0(T_0, T_{1,}.....) + \varepsilon^1 q_1(T_0, T_{1,}.....) + \varepsilon^2 q_1(T_0, T_{1,}.....) + ...$$

3 RESULTS AND DISCUSSION



Physical properties	Skins	PZT materials Sensor/actuator
Length (m)	$L_b = 0.18$	$L_a = L_s = 0.3$
width (m)	$B_b = 0.025$	$B_a = B_s = 0.05$
Thickness (m)	$h_b = 0.7$	$h_a = h_s = 0.008$
Volumic mass (Kg/m³)	$\rho_m = 9246$ $\rho_c = 2730$	$\rho_p = 9200$
Young's modulus (Gpa)	$E_m = 305,9$ $E_c = 422,2$	$E_p = 65,9$
Constant Deformation (m/V)		$d_{31} = 153 \times 10^{-12}$
stress constant (m/V)		$g_{31} = 11,7 \times 10^{-3}$

Fig 1: bimorph piezelastic/elastic/piezoelectric beam **Tab 1. ¨Physical Properties**

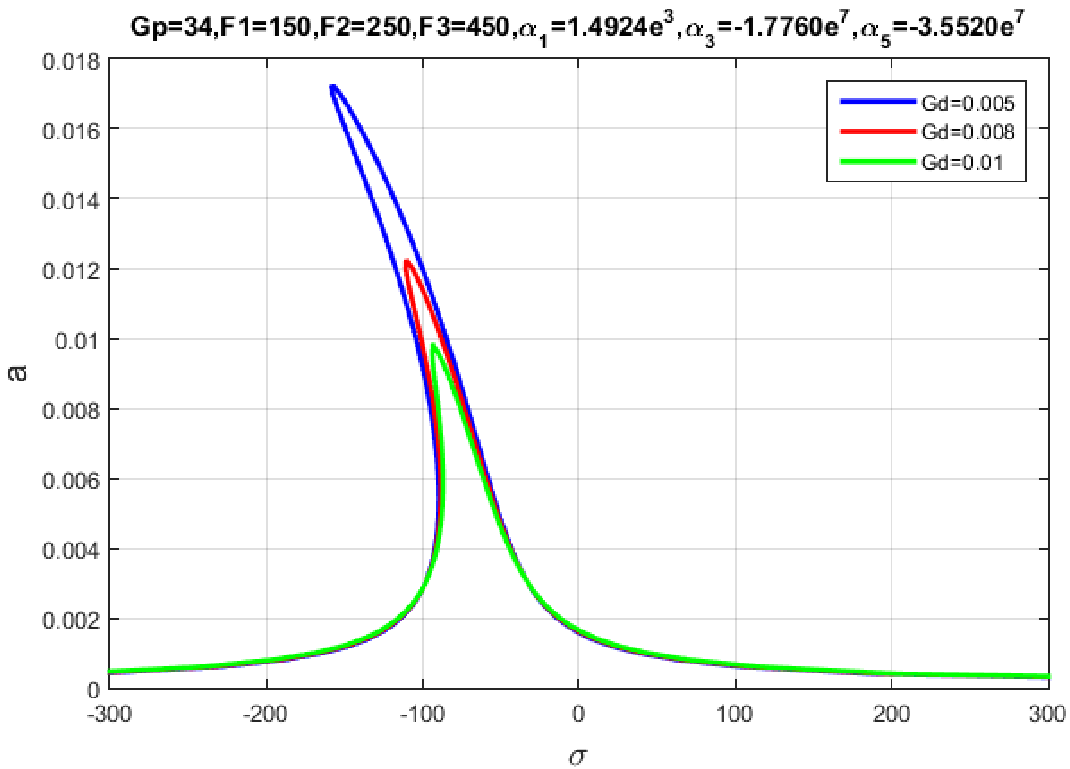


Fig 2: Non-linear frequency-amplitude response for *Gd* (0.005 ;0.008 ;0.01 ; 30) Gp = 34

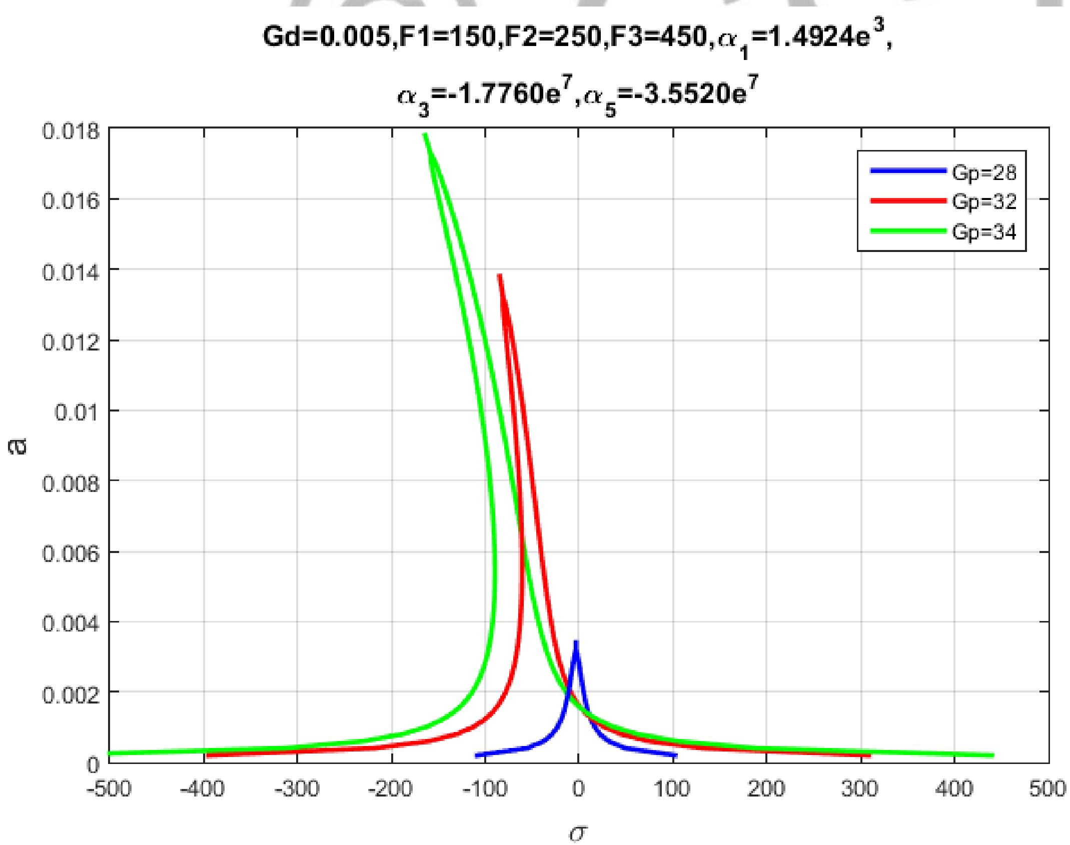


Fig 3: Non-linear frequency-amplitude response for *Gp* (28 ;32 ;34) Gd = 0.005

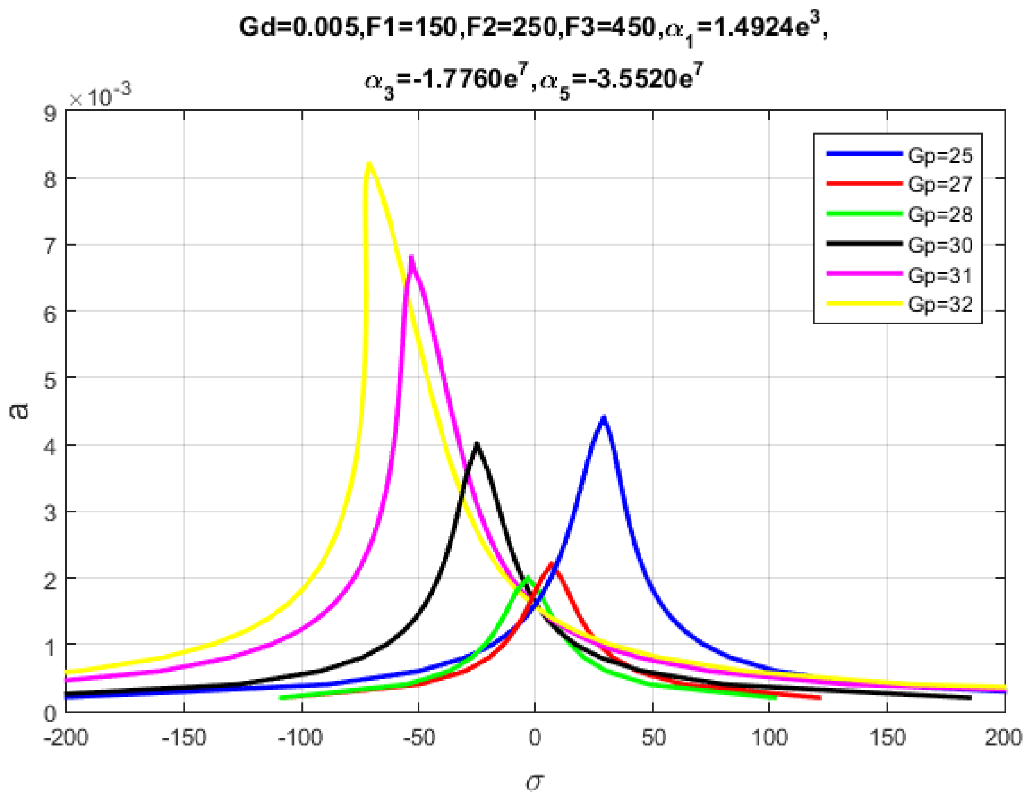


Fig 4: Non-linear frequency-amplitude response for *Gp* (25 ;27 ;28 ; 30 ; 31 ; 32) Gd = 0.005

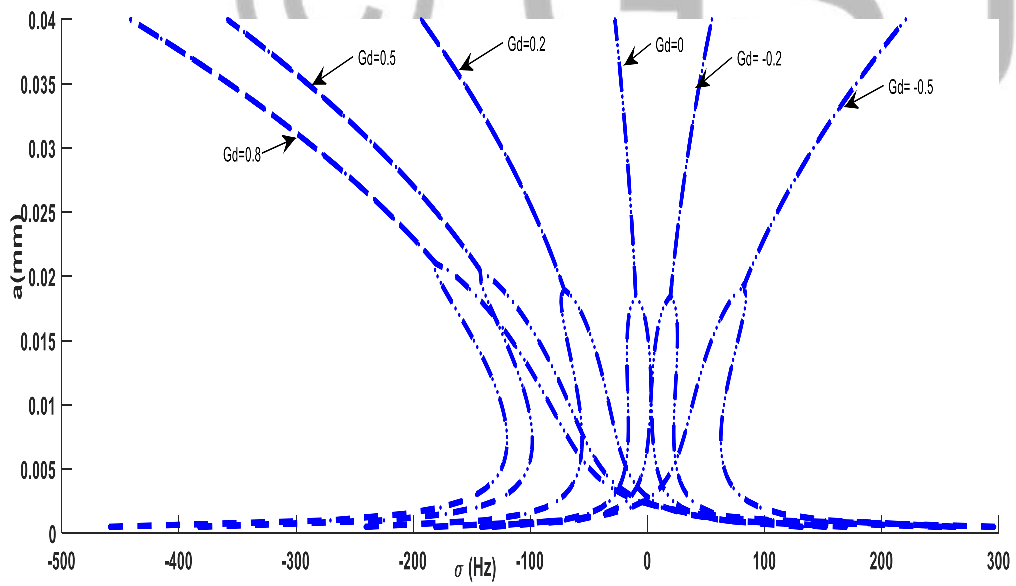


Fig 5: Nonlinear frequency response for *Gp* (-0,5 ; -0,2 ; 0 ; 0,2, 0,5, 0,8) Gd = 30

We can see from these curves that the gain parameters Gd have a considerable influence on the vibration amplitudes. Indeed, for negative values of gain Gd, Gd (-0.5 ; -0.2), the curves obtained are oriented towards high frequencies, which reflects the stiffening nature of the beam. On the other hand, for positive values of Gd (0.2; 0.5; 0.8), the curves are oriented towards low frequencies with relatively large amplitudes, reflecting the beam's softening behaviour. On the other hand, for positive values of Gd (0.2; 0.5; 0.8), the curves are oriented towards low frequencies with relatively large amplitudes, which reflects the softening behaviour of the beam. For the zero value of Gd (0), the curve obtained describes a more linear behaviour, with the amplitude of vibration increasing rapidly, then decreasing just as quickly and stabilising, which would explain why the plate no longer vibrates. We can see that the angles between the different forces applied to the beam play an important role in the behaviour of the structure. Indeed, when the difference in angle is zero, the curves obtained are practically grouped together and, depending on the values of Gd, the beam may stiffen or soften. Figure 4 shows the frequency-amplitude behaviour of the beam as a function of the parameter Gv and the angles between the different forces. It can be seen that the amplitude decreases with Gv. The curves have the same orientation and are symmetrical about the same axis. Our results can be explained by the fact that the greater the gain parameter Gv, the more the vibration amplitude decreases. On the other hand, for small values of Gv, the amplitude is large. The curve shows the amplitude-frequency curves of the non-linear vibrations of a beam on which a direct proportional control has been carried out with Gd as the control parameter. It can be seen that the amplitude of the vibration decreases as the values of Gd become larger, and also that the amplitude of the vibrations decreases with the points where the forces are applied; in fact, the further these points are from the origin of the reference frame, the greater the amplitude of the vibration.The same amplitudes can be observed whatever the gain parameter Gd. This could lead us to say that the points of application of the forces play a predominant role in the frequency responses. We can also see that the curves are oriented towards high frequencies, depending on the values of Gd , where the beam acquires a stiffening or resisting character. In this case, the beam vibrates more for longer, which makes it less stable. The curves illustrate the non-linear behaviour of the sandwich beam under the influence of the velocity gain Gv, we can see that the curves have the same orientation and are symmetrical about the same axis. The amplitudes evolve in the opposite direction to the parameter Gv, i.e. the amplitudes decrease with the values of Gv.

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