



Mathematical Thinking Transformation and Children's Adaptive Anxiety

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Abstract:

Adaptive anxiety in the process of mathematical thinking transformation is a psychological difficulty and thinking gap that students face throughout the construction of their mathematical thinking. Tension and pain are frequently present. Adaptive anxiety will also fade away after they leave the maths learning scenario. The shift of thinking, the appearance of complicated learning problems, and the breaking of the knowledge chain are the major causes of adaptive anxiety in primary school kids. It must be resolved by educating students about the true root of their worry, teachers' reasonable use of transformation tools, and textbook content organization. Reduce elementary school children' adaptive anxiety during the thinking transformation process and improve their physical and mental wellbeing.

Keywords: Adaptive anxiety; mathematical thinking; thinking transformation:-

Objectives and Background:

Students in primary school experience various knowledge and cognitive transitions, including the conversion of physical things to numbers, integers to decimals, positive numbers to negative numbers, and arithmetic to algebra. . These transformations are essentially the transition from one kind of thinking to another with the help of specific mathematical content. It , involves primary thinking types such as perceptual action thinking and concrete image thinking, but also extensively involves

abstract thinking, logical thinking, dialectical thinking, and other advanced thinking types. The physical and mental anxiety of primary school students in switching between many different thinking types is called adaptive anxiety. Adaptive anxiety is the psychological challenge and thinking gap encountered by students during the formation of mathematical thinking. A sense of tension and discomfort often accompanies it. Once they leave the mathematics learning situation, the anxiety will also disappear. Therefore, adaptive anxiety is usually manifested as state anxiety. As a result, adaptive anxiety commonly emerges as state anxiety, causing memory resources to be crowded out, thinking growth to be impeded, and arithmetic learning performance to be impaired in children.

The manifestations and effects of adaptive anxiety:

The inconsistency in thought transformation is the source of adaptive anxiety in mathematics learning—adaptable anxiety results from thinking transformation, while adaptive anxiety results from thinking transformation. **Typical form of adaptive anxiety:**

Mathematics is one of the compulsory subjects for primary school students. At this stage, students are acquiring different knowledge and a transition period of different thinking—including the transition from concrete image thinking to abstract logical thinking, and the transition from arithmetic thinking to algebraic thinking. The first conversion step is more typical in elementary school's lower grades. Physical items are used to introduce students to maths. Teachers will use a finger, a bean, a pen, and other specialized things to depict the number one. When the lower grades learn to add $2+3$, the teacher will take out 2 pieces of chalk, and then take out 3 pieces of chalk, and the students will quickly come to the final result of 5 according to the number of pieces of chalk. Students will feel uneasy if teachers hide the actual items that symbolize numbers. The transition from concrete visual thinking to abstract logical reasoning is time-consuming and challenging. Students require time to adjust to abstract number cognition and learn from the evolution of knowledge and real-life experiences. Except for natural numbers, German mathematician Kronecker feels that everything else is artificial and does not adhere to the criteria of pupils' cognitive growth. For example, most primary school pupils find negative numbers inaccessible and difficult signs. When solving linear equations, ancient

mathematicians frequently encountered subtracting big integers from decimals, according to "Nine Chapters of Arithmetic." Negative numbers were developed to solve the equations and fulfill the closure of this subtraction computation. It is artificially created based on knowledge and needs within mathematics, not naturally generated. Therefore, negative numbers are for elementary school students with basic life experience and those. Negative numbers are considered absurd by some mathematicians in Europe because of their diverse applicable conditions and peculiar expressions. The French mathematician Veda rejected the concept of negative numbers and directly avoided negative numbers during operations, believing it to be nonsense and a nihilistic minus zero [1]; French mathematician Stiefel thought that 0 minus 4 was pure nonsense.

Therefore, learning negative numbers requires an adaptation period, understanding the closed nature of subtraction calculations, and understanding why negative numbers are generated and how to apply them. Teachers need to teach according to the mathematical thinking characteristics of primary school students. Suppose they only explain the concept and application of negative numbers to students, and let students passively accept abstract concepts and algorithms. In that case, it will lead to long-term confusion and anxiety in students.

Adverse Effects of Adaptive Anxiety:

Anxiety is a comprehensive negative emotion, an emotional experience dominated by tension and accompanied by anxiety in the face of threats and inability to do anything. Adaptive anxiety, on the other hand, will occupy students' working memory space, interfere with their thinking growth, raise their cognitive load, and, as a result, hinder their mathematics learning performance. According to Eysenck and Calvo's [2] processing efficiency hypothesis, "anxiety caused by a person will consume a portion of the limited working memory resources, resulting in a reduction in the resources required for cognitive operations, and processing efficiency would subsequently decline." "The emergence of adaptive anxiety leads to weakened learners' processing efficiency, and the low processing efficiency leads to increased memory load, further interfering with task performance, especially in tasks involving arithmetic language sensory memory; the higher the anxiety level, the worse the performance. This view is confirmed by the research of Ashcraft and Kirk [3].

In natural numbers, the multiplication structure with the expression $ab=c$ has some common aspects with the addition structure - multiplication is a repeated addition. Still, it also has its particularity and cannot be reduced to an addition operation. For example, if Xiao Ming drives 38 kilometres in 54 minutes, how many kilometers will he drive in 27 minutes? Students of different ages have a hard time distinguishing between multiplication and addition. This difficulty manifests in students overusing the wrong addition method in the multiplication situation. O manifests in students overusing the wrong addition method in the multiplication situation and overusing the wrong method in the addition situation [4] These research results show that the transformation of students from additive thinking to multiplication thinking manifests students' higher-order thinking development. The learning of multiplication requires students to think from multiple dimensions. The single-dimensional characteristics of primary school students' mathematical thinking make them adapt to this situation. The stage is at a loss [5], which leads to anxiety. In addition, from the learning of arithmetic to the learning of algebra, the increase of the difficulty of knowledge, but also the increases of the difficulty of knowledge and the increases of the difficulty of knowledge and the change of cognitive structure the transformation of thinking. When primary school students learn arithmetic, they mainly use the thinking structure formed naturally in mathematics learning, that is, the natural structure. In contrast, algebra learning needs to use the processing structure corresponding to the natural structure, that is, the should structure. The difference between the two structures constitutes the Difficulties in converting arithmetic to algebra [5], causing adaptive anxiety.

An Analysis of the Causes of Adaptive Anxiety

The causes of adaptive anxiety in primary school students are extremely complex. Based on the characteristics of mathematics, this paper discusses the causes of adaptability from three aspects: discomfort caused by thinking transformation, cognitive load caused by unfamiliar knowledge, and fragmented knowledge caused by knowledge chain breakage and the mechanism of anxiety.

Changes in thinking lead to students' sense of incompatibility:

According to the Compulsory Education Mathematics Curriculum Standard (2011 Edition), mathematics textbooks contain a variety of thinking and their

transformations. The most common and very important transformation is arithmetic thinking and algebraic thinking. The lower grades (grades 1-3) mainly focus on arithmetic thinking, supplemented by algebraic thinking, while the upper grades (grades 4-6) are the opposite. The content of the two is very different. Specifically, it includes the following aspects: First, arithmetic thinking is mainly the operation and splitting of logarithms, and algebraic thinking is mainly the operation and transformation of algebraic expressions. Students take a long time to adjust and experience anxiety when studying algebra because of the fundamental differences in the operational objectives of the two. The essence of learning mathematics is the progressive and re-learning of thinking, and the appropriate meaning of elementary school pupils' learning is the ongoing construction of thinking. Second, arithmetic thinking is procedural, and focused on quantity computation. In contrast, algebraic thinking is structural, focusing on the symbolization of relations and their operations, has analytical qualities, and may obtain the final result using relation symbols.

Answer

Third, arithmetic thinking is mainly based on solving practical problems, starting from specific situations and not universal. While algebraic thinking is situational in solving problems, it is general and relies on logic. Fourth, arithmetic thinking is reverse thinking. Starting from the result, the answer is reversed, and only known numbers are allowed to participate in the operation, and then the final result is pushed forward step by step; while algebraic thinking is a kind of forward-thinking, in algebraic thinking. The status of the known and unknown numbers is the same, and the unknown number participates in the whole operation. and unknown numbers Therefore, the conversion from arithmetic to algebra is a challenge for students. The key to this cross-domain lies in the systematic application of the operational laws of the number system [1].

Students can use one of three techniques to solve the problem of hens and rabbits in the same cage in fourth grade mathematics book II: one is to sketch a visual representation of how many chickens and rabbits there are; the other is to guess. Using the information provided by the question stem, make educated predictions and assumptions. The third option is to use the table technique, which entails listing them in a table until you locate one that answers the query. These methods blend arithmetic

and reasoning and are fundamentally the same, reflecting different assumptions. Students have not yet recognized the grandeur and distinctiveness of algebraic reasoning. They believe that solving difficulties with numerical reasoning is more successful. This is clear and straightforward, demonstrating that kids' minds are strongly established in arithmetic reasoning. Furthermore, according to the poll, nearly half of the students in the assignment "representing numbers with letters" will make mistakes in their application of this information. The quantitative relationship is still largely a problem of conversion between arithmetic and algebraic reasoning..

In short, the value of "algebra" teaching is to provide students with a new way of solving problems, but also to provide students with a new way of solving problems and cultivating students' logical thinking. so that students can master the process of learning from intuitive observation to abstract description to logic and Mathematical method of operation. Piaget's cognitive development theory points out those students at the age of 7-11 are not able to carry out logical abstract thinking well. The reasons are the heterogeneity of different mathematical knowledge and thinking types and the students' equivalence.. The understanding of the relationship is not deep enough. In the dilemma of thinking conversion, many students cannot adapt to the understanding and application of new knowledge and thinking. They will gradually resist and escape from algebra, which will lead to adaptive anxiety.

Complex learning tasks lead to cognitive load on students:

In addition to the dilemma of thinking switching, cognitive load theory also provides a perspective and theoretical basis for the generation of adaptive anxiety. "Students who encounter complex learning tasks will still have obvious cognitive difficulties" [6]. Psychologist John Sweller believes that this is due to the limited working memory capacity and attention to the selection of information, thus proposing the cognitive load theory [7]. When individuals embark on complex learning tasks, they need to process multiple problem-solving information factors in their working memory simultaneously, which may overload the working memory with 7 ± 2 chunks, and cannot process information effectively. Anxiety. Compared with low cognitive load, high-anxiety individuals performed worse under high cognitive load conditions, manifesting as longer search latency, whereas low-anxiety individuals did not show this effect [8].

Because the required knowledge of fractions cannot be immediately retrieved, students must assess the magnitude of the denominator and whether it can be directly carried out while performing fraction operations as they progress from integers to fraction operations. Consider the guidelines for about points and common points, as well. Working memory capacity is, however, limited among pupils. They will require a significant amount of time because these knowledge and regulations are unfamiliar. When students who had previously been exposed to fractional learning were given the problem $3/5 - 2/3$, the majority of them received a $1/2$. Because integer arithmetic has such a stronghold on children, it takes time to adjust to their first encounter with fractions. Fractions are a more demanding learning job than integers, resulting in an increased cognitive burden for students.

Therefore, the complex and diverse knowledge systems in the textbooks, the difficulty of mathematical knowledge, and the complexity of fraction operations occupy students' limited working memory space, causing students' intrinsic cognitive load and anxiety during this adaptation stage.

The broken knowledge chain leads to the fragmentation of students' knowledge:

The structure of children's psychological development mainly involves assimilation and adaptation, which are two forms of adaptation. In learning, children always use the original cognitive structure to assimilate new knowledge. If the schema level of the new knowledge is lower than the original knowledge, the students' cognitive structure will be temporarily balanced, and the amount of knowledge will vary. a significant increase. On the contrary, students need to adjust the original cognitive structure and change their mental schema to achieve cognitive balance. The learner's mastery of knowledge is achieved in the round-trip process of "balance-unbalance-new balance. However, in this process, if there are errors in knowledge, or it is difficult for learners to assimilate and adapt to the incorporation of new knowledge smoothly, it will cause a break in their knowledge chain and affect the establishment of a new cognitive structure. It can be seen from this that the essence of psychological development is the change and improvement of schema levels from low to high. During this process, students need to adapt. If the high-level schema makes it difficult for students to cross, adaptive anxiety will occur.

Luo Yuhua believes that "the knowledge of decimals and integers that he has learned

before is very different and difficult, which makes him confused and confused, and cannot normally accept the transformation from integers to decimals" [9]. Primary school students are exposed to decimals in the second volume of the fourth grade, and the Ministry of Education textbooks use a ruler to measure the desk to lead out decimals. Decimal involves not only units, tens, hundreds, etc., but also the application of decimal points, tenths, percentiles, thousandths and so on in the decimal part. The amount of memory for these parts is much larger than integers. In addition, in addition and subtraction of decimals, students should pay attention to the alignment rules of decimal points when calculating. If you encounter $8.3-6.45$, how should you align the formulas, and what rules should you pay attention to? The analysis shows that the arithmetic rules of decimals and integers are very different, and the learning of these rules requires time to accumulate and accumulate. Suppose students still use the arithmetic rules of integers to solve decimal operations and confuse the difference between integers and decimals. There will be multiple breaks or faults in the learning of decimals, which will lead to multiple exam failures. Students are anxious. If the confusion of these rules leads to habitual errors, then there will be frequent errors in subsequent decimal addition, subtraction, multiplication and division operations. And the increased frequency of mistakes can make them feel scared and uneasy, thereby deepening adaptive anxiety.

In general, primary school students learning mathematics is to gradually incorporate different types of knowledge into their cognitive framework, and finally cultivate and use different types of mathematical thinking. However, for primary school students, the heterogeneity of different types of mathematical knowledge and thinking is a huge challenge. Many people often feel uncomfortable and anxious in the process of thinking conversion.

Countermeasures and suggestions for solving children's adaptive anxiety:

The adaptive anxiety of primary school students is caused by the dilemma in the process of mathematical thinking transformation. Therefore, this paper believes that the adaptive anxiety of primary school students in the process of mathematics learning should be solved from the following three aspects.

Teachers help students analyze the true source of adaptive anxiety:

Anxiety is related to human psychology. In the process of learning mathematics,

students should recognize the threat of the psychological subconscious and expose it to the level of consciousness [10] to understand their mathematics learning level and which ones are their own. If some shortcomings or problems cannot be solved, make a good record. Only by fully understanding yourself can you get rid of anxiety. Excessive anxiety of learners is not only unhelpful but harmful to mathematics learning. The new round of curriculum reform requires teachers to establish a democratic and equal view of teachers and students, clearly stating that teachers must fully respect each student's right to speak, respect each student's differences, and change the traditional authority status. Professor Feng Jian jun believes that "intersubjectivity transcends the objectified relationship between teachers and students, and moves towards equal communication between teachers and students, which is the progress of the understanding and practice of teacher-student relationship." [11] He fully expressed his respect for students and his respect for life. Therefore, teachers should become supporters and understand of students. Teachers should use patient guidance instead of criticism and accusations for 'students' questions. Students can ask questions more confidently, express their doubts and confusions more comprehensively, and address the problems that may cause anxiety. Record knowledge points and analyze and solve them one by one to prevent anxiety.

Solving Math Problems Using Transformational Strategies:

Transformational thinking is often used to solve mathematical problems, transforming difficult mathematical problems into a more familiar form, and then solving the problem. The famous Hungarian mathematician P. Lusa pointed out: "The thinking process of mathematicians is peculiar. They constantly deform mathematical problems through reverse thinking and turn them into familiar and solvable problems" [12]. The algebraist Leopold believed that other numbers are artificial in mathematics in addition to natural numbers [13]. The appearance of negative numbers is artificially created to make equations solvable. Therefore, negative numbers themselves are the best example of thinking transformation. In learning negative numbers, it is necessary to start from the actual life of students. In addition, before learning algebra, students are exposed to simple arithmetic calculations because arithmetic and algebra calculation methods and logical thinking are different. There are certain difficulties in the conversion between the two types of thinking. Therefore, it is necessary to use transformational thinking when learning algebra. Take the "Representing Numbers with Letters" section in the People's Education Edition "Mathematics" as an example.

It is the initial stage of the transformation from arithmetic thinking to algebraic thinking. The firmness of the foundation directly affects the later stage. of deep learning. Three unrelated materials are used in the layout of the textbook. One is the age of the daughter and father. Some of the laws of operation learned. But do the examples in the book interest the students and get them to grasp the content of this section? Some teachers use the real and interesting case of "WeChat to grab red envelopes" to introduce letters, and use five red envelopes to integrate the knowledge points that three completely different examples in the textbook want to express. Different red envelopes point to different goals. Students are more receptive and at the same time prevent the production of adaptive anxiety.

Reasonable arrangement of textbook content to reduce students' cognitive load:

The arrangement of textbooks should consider the systematic relationship between knowledge and consider the cognitive characteristics and life experience of students. Students' cognitive characteristics and life experience will prompt students to master relevant knowledge points, and the corresponding knowledge will also make students' life experience systematized and systematic. Some scholars mentioned that, looking at the arrangement of geometric knowledge in the elementary school mathematics of the People's Education Edition, we can find there are two problems. Knowledge. Second, the combination of numbers and shapes in the teaching materials is not enough, so that students cannot master the calculation of area and understand the characteristics of the corresponding surface at the same time.

On the other hand, looking at the distribution of geometric knowledge in California textbooks, it can be found that the order of this knowledge point is very different from that of Chinese elementary school mathematics textbooks. The arrangement is completely based on the intuitive thinking of second grade students. Starting from life experience, let students fully understand the knowledge of three-dimensional graphics closely related to life, and then learn the knowledge of plane geometry. This arrangement enables students to smoothly transition from understanding three-dimensional geometry to the abstract thinking of understanding straight lines and curves. Therefore, we can learn the specific operation of this point from the United States, and promote the smooth transition of students' geometric thinking level. In addition, because of the problem that the combination of numbers

and shapes is not closely combined, teachers should fully integrate the relevant knowledge of "numbers and algebra" with geometric knowledge. When teaching relevant geometric knowledge points so that students can have a deeper understanding, and at the same time promote better knowledge conversion and knowledge transition.

References:

- [1] Hu Zuoxuan. What is Mathematics [M]. Beijing: Peking University Press 2008.
- [2] Eysenck M W, Calvo M G. Anxiety and performance: The processing efficiency theory[J]. *Cognition & emotion*, 1996,6(6):409-434.
- [3] Ashcraft M H, Kirk E P. The relationships among working memory, math anxiety, and performance[J]. *Journal of experimental psychology: General*, 2001,130(2):224.
- [4] Misailidou C, Williams J. Diagnostic assessment of 'children's proportional reasoning[J]. *The Journal of Mathematical Behavior*, 2003,22(3):335-368.
- [5] Fan Wengui. Primary School Mathematics Teaching Theory [M]. (Second Edition). Shanghai: East China Normal University Press: 2016.
- [6] Pang Weiguo. Cognitive Load Theory and Its Teaching Implications [J]. *Contemporary Education Science*, 2011, 315(12): 23-28.
- [7] Chen Yan, Luo Zengru, Zhao Jianbin. Mathematical errors from the perspective of cognitive load theory [J]. *Journal of Mathematics Education*, 2009,18(04):19-22.
- [8] Basanovic J, Notebaert L, Clarke PJF, MacLeod C, Jawinski P, Chen NTM. Inhibitory attentional control in anxiety: Manipulating cognitive load in an antisaccade task[J]. *PLOS ONE*. 2018,13(10):1-16.
- [9] Luo Yuhua. Causes and Countermeasures of Mathematical Anxiety in Primary School Students [J]. *Journal of Mathematics Education*, 2008,27(03):100-102.
- [10] Xie Mingchu. Prevention and elimination of anxiety in mathematics learning [J]. *Journal of Mathematics Education*, 2002, 11(1): 43-46.
- [11] Feng Jianjun. Otherness: The Teacher-Student Relationship Beyond Intersubjectivity [J]. *Higher Education Research*, 2016, 258(08): 1-8.
- [12] Liu Jiaxia, Liu Linna. Effective Teaching Mode of Primary Mathematics [M]. Beijing Normal University Press, 2014.
- [13] Jordan. Eilberg. 'Devil's Mathematics [M]. Translated by Hu Xiaorui. Beijing: CITIC Publishing Group, 2015.