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Pavan Gampala's Pattern: A Novel Observation in Arithmetic Sequences

Abstract

In this paper, we present a newly observed pattern in the sums of consecutive natural numbers. The pattern demonstrates that the sum of the first n natural numbers, when added to the square of n, equals the sum of the next n natural numbers. This finding introduces a unique relationship within arithmetic sequences, offering a fresh perspective on the properties of natural number summation.

1. Introduction

The study of arithmetic sequences has been fundamental to mathematics for centuries. Classical results, such as the formula for the sum of the first n natural numbers, have been well-established. However, new observations can still arise from re-examining these sequences. This paper introduces "Pavan's Pattern," a novel observation that connects the sum of the first n numbers plus the square of n with the sum of the subsequent n numbers.

2. The Pattern

We define "Pavan's Pattern" as follows:

Given a natural number n, the sum of the first n numbers plus n^2 equals the sum of the next n numbers.

Formally, let S1(n) be the sum of the first n natural numbers:
$S1(n) = \Sigma k=1 \text{ to } n\ (k) = n(n + 1)/2$

The observation states:
$S1(n) + n^2 = S2(n)$

where S2(n) is the sum of the next n numbers:
$S2(n) = \Sigma k=n+1 \text{ to } 2n\ (k)$

3. Proof of the Pattern

We begin by expressing S2(n) explicitly:

$S2(n) = \Sigma k=n+1$ to $2n$ $(k) = (n+1 + 2n)n / 2 = (3n+1)n / 2$

Now, consider the expression for $S1(n) + n^2$:

$S1(n) + n^2 = n(n + 1)/2 + n^2$

Simplifying further:

$S1(n) + n^2 = (n(n + 1) + 2n^2) / 2 = n(3n + 1) / 2$

Thus, we find:

$S1(n) + n^2 = S2(n)$

This confirms the pattern for all natural numbers n.

4. Examples

Let's consider $n = 2$:

The sum of the first 2 numbers: $S1(2) = 1 + 2 = 3$

Adding 2^2: $S1(2) + 4 = 7$

The sum of the next 2 numbers: $S2(2) = 3 + 4 = 7$

Thus, $S1(2) + 2^2 = S2(2)$, confirming the pattern.

5. Generalization and Implications

"Pavan's Pattern" highlights a specific property of consecutive integer sums, suggesting potential avenues for further exploration in number theory. The discovery may inspire similar observations in other arithmetic sequences or inspire research into related combinatorial identities.

6. Conclusion

"Pavan's Pattern" offers a fresh insight into the relationships between consecutive sums of natural numbers. While rooted in fundamental arithmetic, this observation opens doors to new explorations in the field of number theory.

References

Classical references to the sum of natural numbers and arithmetic sequences.