

Project Scheduling Techniques: Displaying and analyzing project schedules

Dr. Daketima G. Briggs

School of Business and Management Studies

Botswana Open University

Abstract

This paper proposed that the planning and executing phases of the project life cycle consist of two dimensions: the technical and sociocultural dimensions. Each of these dimensions includes different processes. The current study examined one aspect of the technical dimensions. Specifically, the study examined the scheduling aspect of the technical dimension. This aspect of the technical dimension is of critical importance because successful completion of projects is continuously being challenged due to late delivery of projects. It has been cited in the literature and observed in practice that poor estimation of activity times is one of the causes of late project delivery. This problem is compounded by the uncertainty in the environment in which projects are performed. Therefore, when estimating project activity durations, the impact of the unknown should be taken into account. This is where the use of the Program Evaluation and Review Technique (PERT) approach to time estimation comes into the project scheduling discourse. That said, the main aim of this study is to display, examine, analyze, and evaluate the scheduling aspect of project management. The aim is accomplished by presenting, examining, analyzing, and evaluating the Critical Path Method (CPM) and PERT networks as the two main approaches to scheduling. This is in line with the observation that a schedule is the conversion of a project action plan into an operating timetable. In this study, CPM is used to estimate the project completion times while PERT is used to predict the probability of various project completion times. In the process, three questions were formulated and answered. Tables and figures (AOA and AON) network diagramming models were used for the display while AON model was used for the analysis and evaluation of the project using the one-time and three-points estimating models. The appropriate scheduling approach (either the CPM or the PERT model) to use in a given context was discussed.

Key words: *CPM, PERT, probabilistic, deterministic, network diagramming, critical path.*

Introduction

Projects are becoming the modus operandi in getting things done in many organizations in the increasingly volatile and competitive environment. Burke (2009) refers to accomplishing businesses through projects as “management-by-project”. Considering the traditional

project life cycle, projects must be initiated, planned, executed, and closed (PMI, 2017). The planning and executing phases of the project life cycle primarily involve two dimensions. These two dimensions are the technical and sociocultural dimensions (Briggs, 2022; Larson & Gray, 2014). Larson and Gray (2014) further observed that the sociocultural dimension of the project management process includes the following: leadership, problem-solving, teamwork, negotiation, politics, and customer expectations. They maintained that the technical dimension of the project management process includes the following: scope, Work Breakdown Structure (WBS), schedules, resource allocation, baseline budgets, and status reports. After the initial high-level plan in the initiating phase of the life cycle, a detailed plan must be developed in the planning phase of the life cycle. In this respect, Burke (2009) describes a project life cycle as a structure having a beginning and an end, with a number of distinct phases in-between. This is understood to be in preparation for the execution and closure stages of the life cycle.

Graham and Portny (2011) noted that the project plan is a high-level plan for the delivery and closure stages, even though the closure stage may be tentative at this point. This is because, the project plan sets down what the project will deliver, how long it will take, activities to be performed, resources required to carry out the activities, and how much it will cost to complete the project (ibid.). This study will examine one aspect of the technical dimension of the project management process. Specifically, the study will examine the scheduling aspect of the technical dimension.

The International Monetary Fund (2012) described Africa as one of the most promising areas in the field of project and program management with regard to long-term development (GDP growth) and research and education dynamism. However, this promise is continuously hindered by the late delivery of projects and programs as “time-to-market” which is of critical importance in project and program management is being continuously challenged. While projects are seen as a means to achieve competitive advantage in Africa, Rwelamila and Purushottam (2012) lament that project management still remains a Cinderella field. Poor estimation of activity times is often cited as one of the causes of late project delivery. This problem is compounded by the uncertainty in the environment in which projects are performed. Therefore, when estimating project activity durations (a scheduling process), the impact of the unknown should be taken into consideration. This is where the use of the Program Evaluation and Review Technique (PERT) approach to time estimation comes into the project scheduling discourse. Hence the focus of this body of work.

Given the above background, as has been alluded to earlier, the aim of this study is to examine and evaluate one aspect of the technical dimension of the project management process. Specifically, the study seeks to examine, analyze, and evaluate the scheduling aspect of the planning phase of the project life cycle. One of the reasons projects usually fall behind schedule is the flawed estimation of activity durations. The project schedule itself, connects the scope, work estimates, and deadline into a Network of tasks [organized logically] (Rao, Gandhi, & Rathod, 2013). In this regard, this study focuses on the estimating and scheduling [time and duration] aspects of the project. Larson and Gray (2014) postulate that “A schedule is the conversion of a project action plan into an operating timetable.” Therefore, the overarching purpose of the study is to use the Critical Path Method (CPM) model to estimate project completion time and to use the PERT model to predict the probability of various project completion times. It is known that PERT and CPM are the most commonly used approaches for project scheduling (ibid.).

There are various methods for displaying and analyzing project schedules. For displaying project schedules, tools such as Milestone Chart, Gantt Chart, and Network diagram, among others are used. For analyzing project schedules, methods such as CPM, PERT, Critical Chain, Resource Leveling, and Schedule Acceleration, among others are used. However, in this work the focus is on Network diagramming, CPM, and PERT usage. The stated methods are often broadly classified as probabilistic and deterministic scheduling techniques (Briggs, 2017). In this respect, CPM is considered to be deterministic in nature and PERT is considered to be probabilistic in nature (Rao, Gandhi, & Rathod, 2013). However, in 2005 the Project Management Institute (PMI) changed the names of these techniques. According to PMI, PERT is called ADM/PERT (Arrow Diagramming Method) and CPM is PDM/CPM (Precedence Diagramming Method) (PMI, 2008). Given the above background and the stated purpose of the study, this study will be guided by the following objectives:

Research Objectives

1. To use CPM and PERT models to develop, display, and analyze a project schedule.
2. To use PERT technique to predict the probability of various project completion times.
3. To determine when a particular scheduling technique is appropriate to use in a given project context.

In order to accomplish the above objectives, the following questions are formulated:

Research Questions

1. How can CPM and PERT models be utilized to develop, display, and analyze a project schedule?
2. Can the PERT technique be used to predict the probability of various project completion times?
3. When is it appropriate to use a particular scheduling technique in a given project situation?

Because of the nature of activities in projects and their relationships, diagramming techniques are often used to provide insights that are not available from raw project data. Network diagrams are some of the tools used to provide insights based on relationships that may not readily be evident in texts. In this respect, Pritchard (2015) noted that network-based schedules... generate graphics that depict the project's activities and their relationships [predecessors, successors, and parallel tasks more visibly].

ADM and PDM for developing, displaying, and analyzing the project schedule: General

Sequencing the work and scheduling the work are related planning activities both of which are necessary to produce a schedule for the project (Howes, 2001). Howes further postulates that the diagrams that project team members use to assist in discovering the sequence of work have historically fallen into two categories namely, the arrow diagramming method (ADM) and the precedence diagramming method (PDM). Fox and Waladt (2007) assert that the two most common forms of network diagramming techniques are the Program Evaluation and Review Technique (PERT) and the Critical Path Method (CPM). Table 1 contains a list of activities, predecessors, and estimated durations. The data in this table are used to construct the two main scheduling diagrams. Therefore, projects can be presented as an activity-on-arrow (AOA) network and as an activity-on-node (AON) network diagram. Similarly, these two widely used project scheduling approaches are also referred to

as the Program Evaluation and Review Technique (PERT) and the Critical Path Method (CPM) respectively. Some practitioners also include the precedence diagramming technique as a third method to present their project (op cit.). However, for the purpose of this study, the project network will be presented as an activity-on-node network even in PERT presentation. In the meantime, the following table (Table 1) contains the inputs used to construct the sample AOA (Figure 1) network and AON (Figure 2) network diagrams respectively. It should be noted that PDM is an AON network model that easily allows for leads and lags within the network (Meredith, Shafer, Mantel, & Sutton, 2014).

Table 1. List of Activities, Immediate Predecessors, and Estimated Durations

Using PERT and CPM approaches to display the ADM and PDM network diagrams.

Activity	Immediate Predecessors	Estimated Duration (days)
A	-	3
B	-	5
C	A	2
D	B	7
E	B	4
F	C, D	9

Table 1 contains the data for displaying the AOA or ADM (Figure 1) and AON or PDM (Figure 2) Network Diagram Examples

Next, the data in table 1 is used to construct activity networks using the AOA and AON techniques respectively.

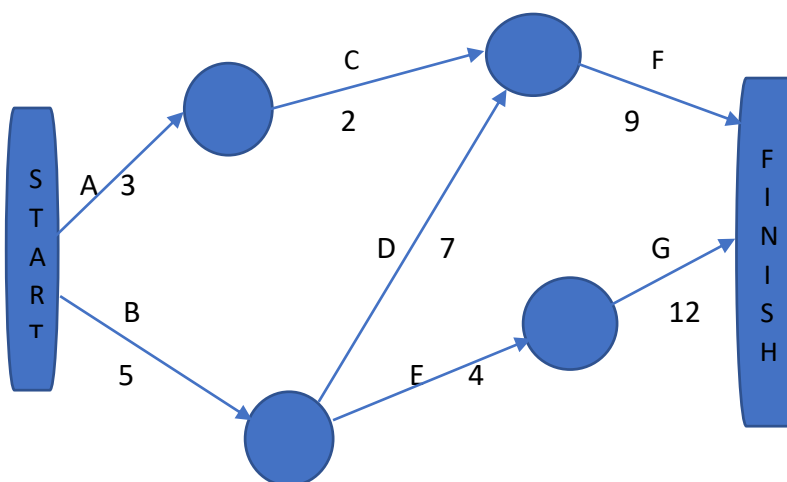


Figure 1: An AOA Network Diagram Example

The AOA is usually associated with PERT network and AON is usually associated with CPM (Meredith, Shafer, Mantel, & Sutton, 2014). Both AOA and AON can be used as scheduling tools. According to Meredith, Shafer, Mantel, and Sutton (2014), a schedule is the conversion of a project action plan into an operating timetable. Such timetable would contain the id,

activities, estimated duration, start date/time, finish date/time, and the relationships (successor, predecessor, parallel activities) between/among the activities, and others.

In figure 1, the events presented as cycles are connected with arrows. The letters on the arrows represent the activities and the numbers below the arrows represent the estimated durations of the activities. It is a seven activities network. A start and finish events (Milestones) are used at the beginning and end of the diagram in order to complete the networks in both figures 1 and 2. Clayton (2014) defines a milestone as a fixed point in the project schedule when something has happened – usually the completion of a task or the creation of a deliverable. He describes type 1 milestones as the big points in the project used to start planning and they mark significant points along the way. Type 2 milestones, he continues, are smaller achievements that are used to track progress and so give an indicator of how well the project is keeping to schedule.

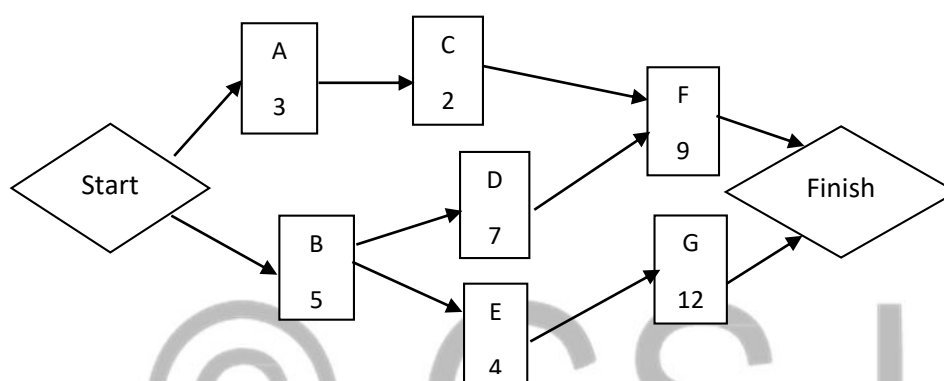


Figure 2: An AON Network Diagram Example

Figure 2 is an AON network diagram. In this network, rectangles (boxes) are used to represent the activities (A-G). The numbers in the rectangles (nodes) are the estimated durations of the activities. This is also a seven activities network as in the AOA network diagram. Both figures 1 and 2 network diagrams are constructed with the same data values.

The project execution phase is preceded by the planning phase in the project life cycle (PMI, 2017). In fact, the planning process is the foundation of all that follows. This observation was supported by Rao, Gandhi, and Rathod (2013) who expressed that the project plan forms the basis for all management efforts associated with a project. The output of the planning process is the project plan (PMI, 2017). The project plan implies a schedule just as a schedule implies a plan (Meredith, Shafer, Mantel, & Sutton, 2014; Larson & Gray, 2014). Given this connotation, one can equally say that planning and scheduling are two sides of the same basic process. Furthermore, it is noted that the project schedule is simply the project plan in an altered format (ibid.). Pritchard (2015) argued that the value of the network diagrams lies in their ability to depict relationships among activities and to provide a clear understanding of how the project will evolve as an integrated whole.

Project Schedule and Project Scheduling

Project schedules and project scheduling are presented differently by different scholars and practitioners. Vaidyanathan (2013) describes a project schedule as the delivery of a project scope. He further describes scheduling as a plan to implement a project using an ordered sequence of activities with time allotted for each activity. It should be noted that in practice, the notion of orderliness is not always maintained. Nevertheless, Mouhoub and Benhocine (2016) describe scheduling as the determination of the timing of the activities comprising

the project to enable managers to execute the project in a timely manner. In this respect, the importance of project schedules cannot be overemphasized. In support of this notion, Kerzner (2013) argues that the primary objective of the development of a schedule is usually to coordinate activities to complete the project with the best time, least cost, and least risk. A project manager who accomplishes his/her project within these three constraints will be deemed a successful project manager. For Ssempebwa (2013), Scheduling is the process of arranging, controlling and optimizing work and workloads in a production process or manufacturing process. On the other hand, he describes a schedule as a basic time-management tool, consisting of a list of times at which possible tasks, events, or actions are intended to take place....

Using PERT and CPM in Analyzing the Schedule: General – CPM

CPM is a schedule network analysis tool used to determine the amount of schedule flexibility (float/slack) on various network paths which determine the minimum project duration. Activities on the critical path drive the finish date of the project (Meredith, Shafer, Mantel, and Sutton, 2014). By identifying the activities with slacks/no slacks, one can determine where potential delays are likely to occur. With a visual presentation and a step-by-step process, the procedure enables one to identify bottlenecks in the project that need to be attended to. The CPM is useful in scheduling, monitoring, and controlling projects. It is the most widely used scheduling model in the industry (Graham & Portny, 2011). By highlighting the activities with slack and float times, one can determine the minimum time required to complete the project. The CPM model provides a one-time estimate for each activity. This provides an actual timeframe for the activity with a high degree of certainty. That is to say in this model, activity estimates provide a discrete number. This method facilitates comparison between actual performance against baseline data. Furthermore, CPM is deterministic in nature (Meredith, Shafer, Mantel, & Sutton, 2014).

Ssempebwa (2013) maintains that CPM provides a way for project managers to determine which activities/tasks are critical (zero slack time) and which activities/tasks can be delayed and for how long. The CPM model has been in use since the 1950s. Graham and Portny (2011) support Ssempebwa's observation saying that the critical path is the longest chain of dependent activities going through the activity network and that it can be seen clearly because critical path activities have a float of zero. These observations are important in project management because, if a critical path activity is delayed, the longest path (CP) gets longer and as a result, the entire project falls behind schedule.

Furthermore, it is safe to say that the project completion date is driven by the list of connected activities having the longest path. In this example, the project duration, start and finish times are calculated from the critical path (CP) activities. Therefore, the focus is majorly on how the project is performing on the critical path activities, rather than focusing on all activities. It should be pointed out that estimations in deterministic scheduling requires experience and reference to past data. Moreover, the chances of successfully completing the project as per the schedule, greatly depends on the estimations that are deterministic in nature. In this respect, one can say that CPM is an activity-oriented technique that is deterministic in nature and uses a one-time activity estimate.

In this model (i.e., CPM model), values such as duration, start and finish dates (or times) for activities are deterministic in the sense that each activity is allocated a single value

estimation. The estimated values roll up to the project level to determine project duration along with start and finish dates or times. The statistical tools generally used in AON network is the Critical Path Method (CPM). Gido, Clements, and Baker (2018) describe the CPM simply as a type of network planning technique.

Program Evaluation and Review Technique (PERT)

PERT takes network analysis a step further by embedding multi-data-point duration estimates to establish [activity] values for schedules (Pritchard, 2015). Just as a schedule compression technique can be used to shorten a project duration without reducing the project scope, a scheduling technique can be used to estimate a project duration without adversely affecting the project scope.

Using PERT analysis enables one to display a chart and/or network diagram with the tasks involved in completing the project as well as identifying the minimum time needed to complete the total project. Besides, PERT chart enables one to visibly display dependencies [precedence relationships] between tasks in the work breakdown structure (WBS); facilitates identification and makes visible the critical paths; identifies the early and late starts and finish dates or times; and identifies the slacks/floats for each activity. Furthermore, PERT enables one to compress project schedules due to a better understanding of dependencies leading to improved overlapping of activities and tasks where possible. Schwalbe (2011) describes PERT as a network analysis technique used to estimate project duration when there is a high degree of uncertainty about the individual activity duration estimates.

Hobbs (2009) describes a dependency as the relationship between two tasks. He further states that dependency can be based either on logic or on resource. There are four types of dependencies: Finish to Start (FS), Finish to Finish (FF), Start to Start (SS), and Start to Finish (SF) (Schwalbe, 2014; Bucki, 2007; Brown & Lyer, 2010; Bunin, 2012). It should be noted that in PERT only FS relationship type is used while in CPM, all four types of relationships can be applicable. Among these types, FS is the most common and SF is the least common. In fact, SF dependency is not supported by most current project management software (Gido, Clements & Baker, 2018; Meredith, Shafer, Mantel, & Sutton, 2014; Graham & Portny, 2011).

Rao, Gandhi, and Rathod, (2013) cited establishment of unrealistic deadlines, underestimation of effort, and Predictable and/or unpredictable risks as causes of projects coming in late. In spite of the uncertainty surrounding activity estimating, a project schedule is required to ensure that required project commitments are met and that performance progress toward achieving these commitments can be tracked. In this respect, PERT is probabilistic in nature (op cit.), that is stochastic. PERT is an event-oriented technique that uses three-time estimates

Comparison: CPM vs PERT

Both CPM and PERT provide project management methods that can enable one to plan, monitor, and update the project as it progresses. However, there are many similarities as well as differences between the two methods. For example, both use network diagrams and follow similar logic in their construction. Mouhoub and Benhocine (2016) noted that the development of PERT and CPM was the result of the evolution of project management as a field. Both are used in scheduling individual activities that make up the project. Both are used in scheduling the earliest start and finish times and latest start and finish times for each activity. However, while CPM estimates of activity durations are based on historical

data and experience thus CPM uses actual or one-time estimates. Hence, CPM is deterministic. PERT estimates, on the other hand, take into consideration the uncertain nature of the environment in which projects are performed. This leads to the use of a range for each activity duration estimate. Hence, PERT is probabilistic.

In practice, CPM and PERT are often used together and they both address how long a project will take. It has been observed that more project managers are using both techniques or merging them in their projects (Eby, 2015). Nevertheless, she stated that there are crucial differences between CPM and PERT. She maintained that the most important is that while PERT focuses on controlling time, CPM focuses on cost optimization. These and other distinctions between the two methods are summarized in table 2.

Activity	Predecessor(s)	Estimated Duration (days)
A	None	4
B	A	7
C	A	6
D	B,C	15
E	B	11
F	D	9
G	C	3
H	E,F,G	5

Table 2: Summary of distinctions between CPM and PERT

Source: Kate Eby (April 30, 2015). *The Ultimate Guide to the Critical Path Method*

If one is using a scheduling tool such as MS Project, it would be easy to develop a hierarchy of work known as the Work Breakdown Structure (WBS) and use it in the estimating process. The tool can be used to estimate the dates and cost of each activity which in aggregate calculates the estimated start and finish date along with the total cost estimate of the project. However, in this work, we are only concerned with timeframes, not with costs.

In this section, the logic used to create an activity network is applied. This includes predecessor and successor activities/tasks. Next, a network using activity-on-node (AON) technique is developed and displayed graphically. Thereafter, activity duration estimation is performed. This used the probabilistic estimating technique (PERT). The following table (Table 3) provides the input values used to construct the network diagram in figure 3.

Table 3: List of Activities, Immediate Predecessors, and Estimated Durations

CPM	PERT
Focus on cost optimization	Focus on time control
Used in projects with predictable activities such as construction	Used in projects with unpredictable work such as R&D
Deterministic treatment of time - single outcome	Probabilistic treatment of time - uses three estimates of duration
Can show activities that can be sacrificed if needed to execute project because it distinguishes between critical and noncritical activities	PERT does not provide this information because it does not distinguish between critical and noncritical activities
Enables "crashing," a shortening of the duration of specific tasks while incurring the least extra expenses	Crashing does not apply

Question 1

How can CPM and PERT models be utilized to develop, display, and analyze a project schedule?

Solution for Question 1

A CPM (AON) technique is used to develop the schedule (an eight activities network). The network is associated with the data in table 3.

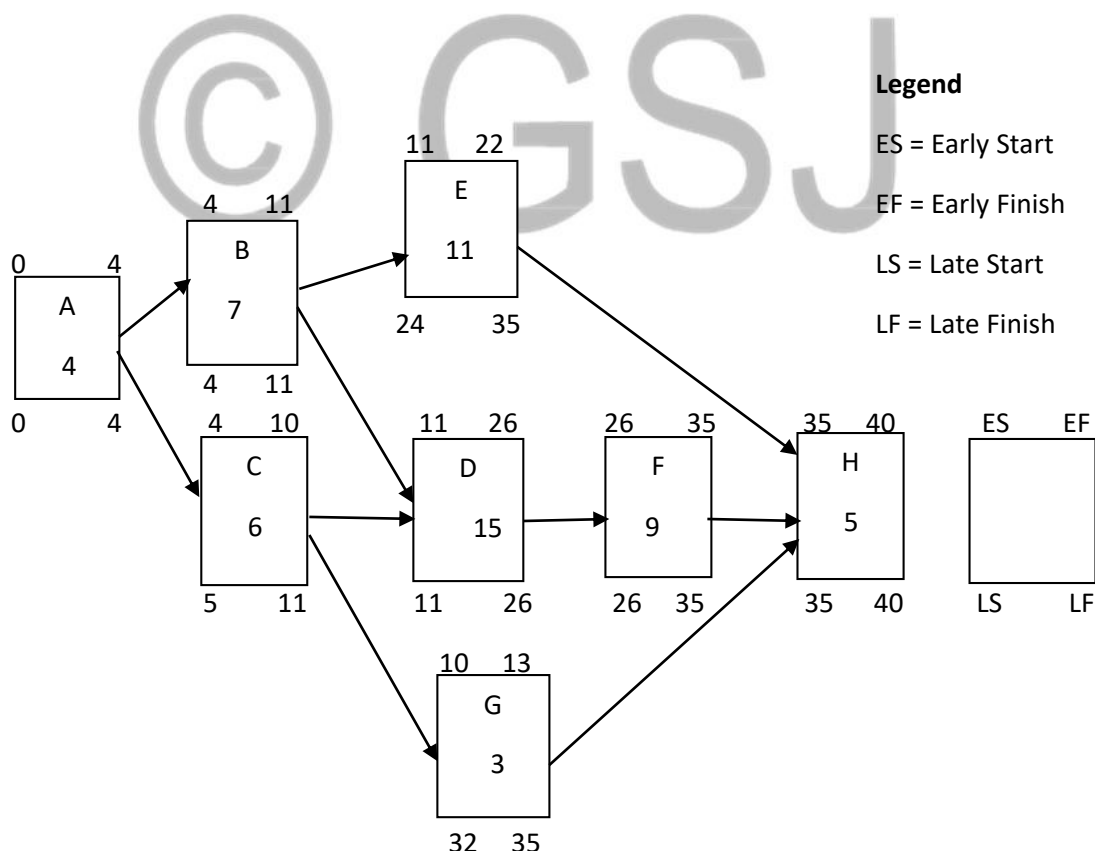


Figure 3: An AON Network Diagram from table 3

In this study, the CPM which is the same as the PDM uses the AON diagramming approach to develop and analyze the project schedule using the data in table 3. Precedence network diagramming is a schedule diagramming technique in which scheduled activities are

represented by nodes and arrows are used to show the logical relationship between activities.

This technique is appropriate because AON networks are used by most of the popular project management software (Meredith, Shafer, Mantel, & Sutton, 2014). Besides, the AON networks are easy to draw since they do not need to use dummy activities (arrows drawn with dashed lines) to complete the network diagram. Even though PERT and CPM have similarities and differences, Meredith, Shafer, Mantel, and Sutton (2014) demand that the project manager should be familiar with both types of networks. Graham and Portny (2011) refer to AON network as the Precedence Network Diagram. The technique of constructing the network diagram is the Precedence Diagramming Method (PDM) we alluded to earlier which uses the CPM technique.

Analysis of the project schedule using CPM technique: Specific

This is an eight activities network shown in figure 3 with data from table 3. Following the Legend on the right hand side of figure 3, A Forward pass is performed first to obtain the earliest times (earliest start (ES) and earliest finish (EF) time). These values are at the top left and right corners of the nodes. Then a backward pass is performed to obtain the latest times (latest start (LS) and the latest finish (LF) time). These are shown at the bottom left and bottom right hand corners of the nodes. The formula used to calculate the EF is as follows:

EF = ES + Estimated duration. The ES of the first activity (Node A) in this case is 0. Therefore, Activity A's EF = $0 + 4 = 4$. Subsequently, The ES of the immediate successor activity is equal to the EF of the predecessor activity. However in this process, if the successor activity is a merge activity (as in activities D and H), its ES is the higher or highest EFs of its immediate predecessor activities. Upon completion of the forward pass (moving from left to right), we perform the backward pass (moving from right to left). We start the backward pass with the LF time of the last activity in the network. The LS and LF times are at the bottom left and right corners of the nodes respectively. The LF of the last activity is usually the same as the EF of the last activity except where there is a different required completion time. If there is a required completion time then, this time takes precedence over the estimated EF obtained through the forward pass process. However, for the purpose of this study, the LF is the EF of the last activity in the network.

Given the above information, in order to do the backward pass, starting from the last activity's LF, we subtract the estimated duration of the activity from the LF to obtain the LS of that activity. The formula used is as follows: **LS = LF – estimated duration.** For example for activity H, the LS is LF – estimated duration which is $40 - 5 = 35$. Subsequently, the LF of a predecessor activity is equal to the LS of its immediate successor activity except where the predecessor is a burst activity (as in activities B and C). In this case, the LF of the predecessor activity is the lower or lowest value among the LS of its immediate successor activities. All activities of all paths must be completed to finish the project. In order to identify the critical path, one needs to determine the longest path (in terms of duration) through the network. Meredith, Shafer, Mantel, and Sutton (2014) note that the shortest time for completion of the network is equal to the longest path through the network.

Figure 3 network has four paths: A-B-E-H = $4 + 7 + 11 + 5 = 27$; **A-B-D-F-H = $4 + 7 + 15 + 9 + 5 = 40$** ; A-C-D-F-H = $4 + 6 + 15 + 9 + 5 = 39$; and A-C-G-H = $4 + 6 + 3 + 5 = 18$. Given that path A-

B-D-F-H with an estimated completion time of 40 days is the longest path and all activities and all paths must be completed to finish the project, path **A-B-D-F-H** is the critical path in this network. If any activity on this path is delayed (even slightly), the project will be delayed. It should be noted that path A-C-D-F-H has a completion time of 39 days. This is near the critical time of the identified critical path; path A-C-D-F-H should be monitored very closely. Any delay on any activity on this path will also make this path a critical path.

When the forward and backward passes are complete, we can calculate the floats or slacks (aka: total float or total slack or simply, float or slack). The formula for calculating the total slack is $LS - ES = LF - EF$. These two formulas will give the same result of the activities on the critical path. Using these formulas, it can be seen that the total slack of every activity on the critical path is zero (0). Therefore, LF must be equal to EF and LS must be equal to ES. In this case, in activity H ($LF - EF = 40 - 40$ and $LS - ES = 35 - 35 = 0$). The same goes to all the other activities **A-B-D-F-H** on the critical path. Further, it should be noted that if LF finishes earlier than EF, the activity will be late, causing a delay in the project. This statement is equally true for LS and ES (Meredith, Shafer, Mantel, & Sutton, 2014). Furthermore, for activities not on the critical path, the LF and EF or the LS and ES will be different. This difference is the total slack/total float (or simply slack or float) for the respective activity.

Analyzing the project schedule using PERT technique: Specific

In the following table (Table 4), activity durations are given in days. Assume the estimates are made at the 99 percent level.

Table 4: Program Evaluation and Review Technique (PERT) Table

Activity	Predecessor	Optimistic Time (a)	Most Likely Time (m)	Pessimistic Time (b)	Expected Time (t_e)	Variance (σ^2)	Std Dev. (σ)
A	None	7	11	15	11	1.8	1.3
B	None	10	10	10	10	0	0
C	A	3	8	13	8	2.8	1.7
D	A	6	11	18	11.3	4	2
E	B	4	7	12	7.3	1.8	1.3
F	B	5	9	16	9.5	3.4	1.8
G	D, E	10	13	20	13.7	2.8	1.7
H	C, F, G	7	12	17	12	2.8	1.7

Solving the Project Network

Schwalbe (2014) asserts that PERT applies the CPM to a weighted average duration estimates. She further notes that PERT uses probabilistic time estimates – duration estimates based on using optimistic, most likely, and pessimistic estimates of activity durations – instead of using a one-time estimate as CPM does. Unlike the CPM as discussed earlier, PERT takes into account, the uncertainty in the activity duration estimates. To use PERT, one needs to calculate a weighted average for the duration estimate of each project activity (Schwalbe, 2014). The following formula from Schwalbe is used in calculating the time estimates using PERT. It should be noted that PERT Network node times are calculated in the same manner similar to AON/PDM times. However, PERT technique uses three time estimates. This view was expounded by Meredith, Shafer, Mantel, and Sutton (2014) stating

that current software handles three-time estimates of duration and can do all calculations almost instantly (p.187). Therefore, given the data in table 4, we need to first calculate the expected time (t_e) and variance (σ^2) for each activity.

Calculating Activity Times – The project in table 4 has three duration estimates for each activity: optimistic (a), most likely (m) and pessimistic (b). Optimistic and pessimistic are defined as the durations that represent 99 percent certainty. In other words, the actual duration of an activity will be less than the optimistic or greater than the pessimistic only one percent of the time (Meredith, Shafer, Mantel, & Sutton, 2014). Then, the expected time (t_e) for each activity is found using the formula:

$(t_e) = (a + 4m + b)/6$. This formula is based on the beta statistical distribution,

- where:
- a = optimistic time estimate
- b = pessimistic time estimate
- m = most likely time estimate, the mode

For example, using the above PERT Weighted Average formula, the (t_e) for each activity is calculated as:

$A = (7 + 4(11) + 15)/6 = (7 + 44 + 15)/6 = 11$. Similarly, $B = (10 + 4(10) + 10)/6 = 10$; $C = (3 + 4(8) + 13)/6 = 8$; $D = (6 + 4(11) + 18)/6 = 11.3$; $E = (4 + 4(7) + 12)/6 = 7.3$; $F = (5 + 4(9) + 16)/6 = 9.5$; $G = (10 + 4(13) + 20)/6 = 13.7$; $H = (7 + 4(12) + 17)/6 = 12$. The (t_e) of each of these activities is recorded in the Expected Time (t_e) column in table 4. These estimates are made at the 99 percent level.

Along with the t_e , the variance (σ^2) of the durations for each activity can be calculated using the following formula:

$$\sigma^2 = ((b-a)/6)^2$$

Therefore, for Activity A, $\sigma^2 = ((15 - 7)/6)^2 = 1.8$ days; $B = ((10-10)/6)^2 = 0$; $C = ((13-3)/6)^2 = 2.8$; $D = ((18-6)/6)^2 = 4$; $E = ((12-4)/6)^2 = 1.8$; $F = ((16-5)/6)^2 = 3.4$; $G = ((20-10)/6)^2 = 2.8$; $H = ((17-7)/6)^2 = 2.8$. The variance is recorded in the σ^2 column in table 4.

The Standard Deviation (σ) of each activity is calculated using the following formula:

$$\sigma = \sqrt{\sigma^2}$$

Therefore, the σ for activity A from table 4 is $\sigma = \sqrt{1.8} = 1.3$; $B = \sqrt{0} = 0$; $C = \sqrt{2.8} = 1.7$; $D = \sqrt{4} = 2$; $E = \sqrt{1.8} = 1.3$; $F = \sqrt{3.4} = 1.8$; $G = \sqrt{2.8} = 1.7$; $H = \sqrt{2.8} = 1.7$. These are recorded in the σ column in table 4 above.

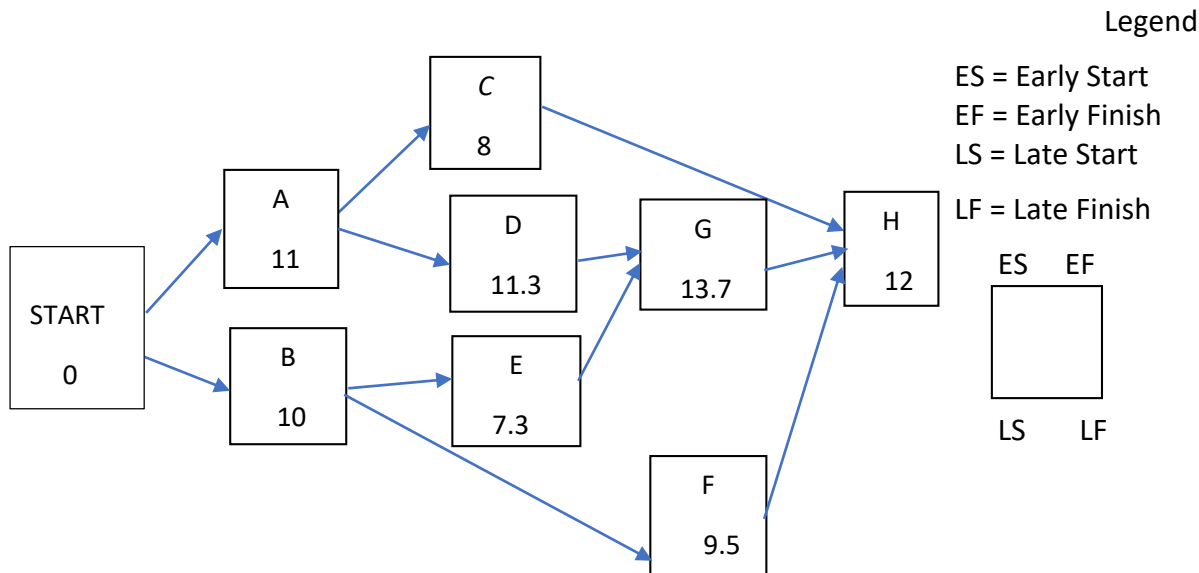


Figure 4: A network diagram with the expected time estimates from column t_e in table 4

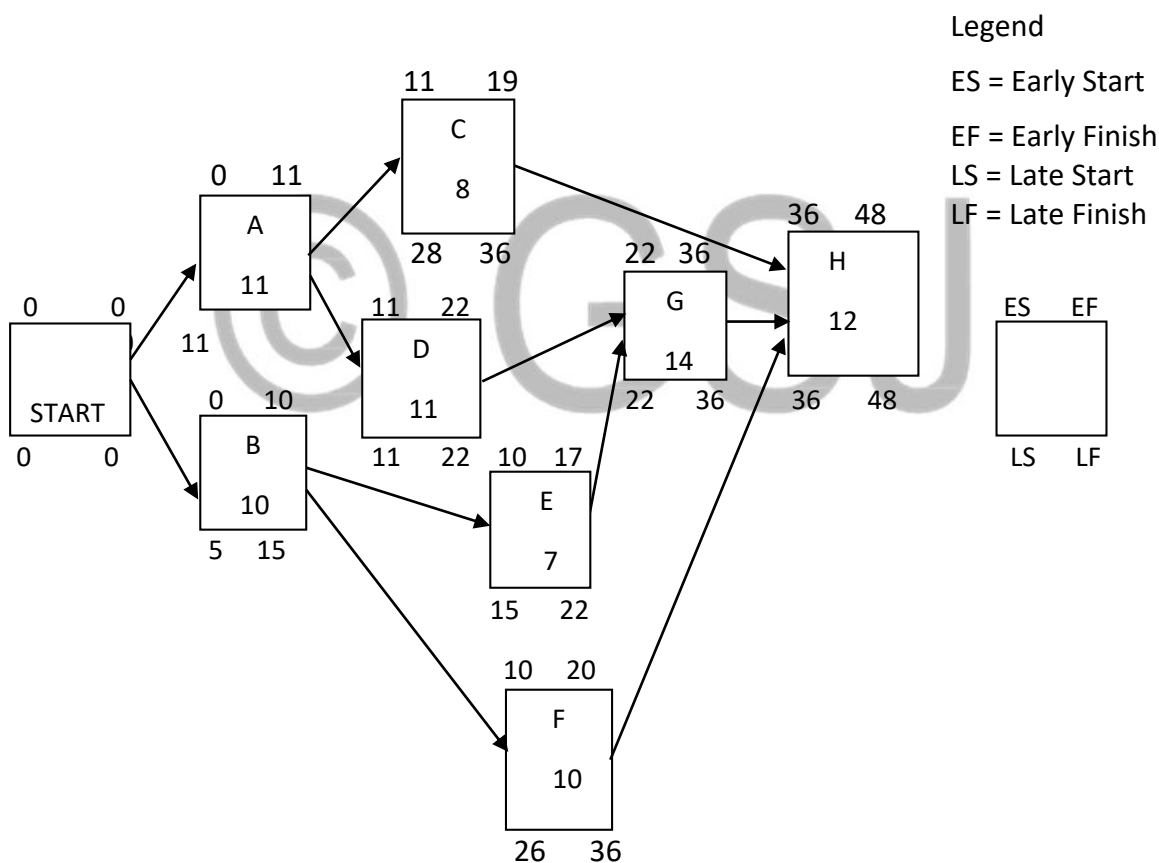


Figure 5: A network diagram with estimated durations from the forward/backward passes

Figure 5 is similar to figure 4. However, in figure 5, the durations are rounded to whole numbers without decimal points in order to simply computations. Besides, figure 5 presents the PERT data in a CPM model using the AON network diagram. Furthermore, upon completion of the forward and backward passes, the network highlights the critical path with the critical activities. In this case, given that path **A-D-G-H** with an estimated completion time of EF and LF of 48 days, it is the longest path and all activities and all paths must be completed to finish the project, path **A-D-G-H** is the critical path in this network.

The computations and procedures are similar to those used in the CPM approach. Therefore, the details of the computations and procedures are not repeated here for determination of the critical path and critical activities. Discussing the critical path, Clayton (2014) provides an illuminating description of “critical path”. He describes critical path as “The longest route through the network chart, representing the duration of the project. It is critical in the sense that these are the activities that carry the risk of delay – if any of these activities were delayed, the whole project would miss its deadline” (p.208). Besides, he describes PERT as “an estimating technique that starts with a network chart and combines optimistic, best estimate, and pessimistic estimates to produce an overall estimate of the most likely duration and standard deviation (spread of likely durations) for a project activity”(p. 212). It should be noted that it is believed that PERT estimates generally produce more accurate estimates than estimates derived from CPM techniques, especially in situations where little or no much is known about the activities.

Using the same formula as in the CPM technique, it can be seen that the total slack of every activity on the critical path is zero (0). Therefore, LF must be equal to EF and LS must be equal to ES. In this case, in activity H ($LF - EF = 48 - 48$ and $LS - ES = 36 - 36 = 0$). The same goes to all the other activities A-D-G on the critical path. Further, it should be noted that if LF finishes earlier than EF, the activity will be late, causing a delay in the project. This statement is equally true for LS and ES (Meredith, Shafer, Mantel, & Sutton, 2014; Clayton, 2014). The path durations are calculated by summing the expected duration of each participating activity. The Project Completion is calculated as the maximum duration of any of the possible paths (48 days in this case).

Question 2

Can the PERT technique be used to predict the probability of various project completion times?

Solutions

The answer is “Yes” and detailed workout follows.

The network associated with the data in table 4 appears in figure 5. Here, however, the values (for t_e) in figure 5 are rounded to whole numbers.

In the network presentation, the critical path is found to be **A-D-G-H** and the critical time for the network is **48 days**. Because the mean time (T_E) is used for all activities on the critical path, there is a 50 – 50 probability of completing the project in 48 days or less – and also, 48 days or more. The calculation is exactly as was done with the CPM technique. It should be noted that a, m, and b are estimates and that durations are ranges, not point estimates. Because the three time estimates for each activity in PERT follow a beta distribution, it is possible to calculate the probability or likelihood of actually completing the project before [or after] the required time (Gido, Clements, & Baker, 2018). This type of calculation (probability calculation), however, cannot be made if only one-time estimated duration is used for each activity as with the deterministic technique. Gido, Clements, and Baker (2018) expanded on this notion. They assert that when three time estimates are used, all of the activities on the critical path of the network diagram can be added together to obtain a total probability distribution. They referenced the central limit theorem of probability theory which states that this total probability distribution is not a beta probability distribution but a

normal probability distribution. This probability distribution is bell-shaped and symmetrical around its mean (or expected) value.

It is against this backdrop that we will proceed to determine the probability of various project completing times. In connection with this, it should be reminded that the expected duration which divides the area under the probability distribution into two equal parts is a measure of the central tendency of a distribution. However, the variance (σ^2) is a measure of the dispersion, or spread, of a distribution from its expected value (ibid.). In an earlier calculation, we used the following formula to calculate the variance for the beta distribution of an activity:

$$\text{Variance} = \sigma^2 = \left(\frac{b-a}{6} \right)^2$$

It is believed that the variance of the total normal probability distribution is equal to the sum of the variances of all the activities that make up the total normal distribution (σ^2). The standard deviation (σ) is equal to the square root ($\sigma = \sqrt{\sigma^2}$) of the variance (σ^2). As was noted earlier, the total probability distribution of all the activities on the critical path of a network diagram is a normal distribution, with a mean or expected value equal to the sum of the individual activity expected durations and a variance equal to the sum of the individual activity variances.

As was stated previously, the critical path of the activities in figure 5 is made up of activities A, D, G, H (**A-D-G-H**) with a critical time (T_E) of 48 days. This is the earliest expected completion time. This is the sum of the individual means, or expected durations. Therefore, there is a probability of 0.5 that the project will be completed before day 48 and a probability of 0.5 that it will be completed after day 48. The reason, according to Gido, Clements, and Baker (2018) is because half of the area under the normal distribution curve is to the left of this expected time; the probability of actually completing a project after its earliest expected finish time is also 0.5, because half of the area under the normal curve is to the right of this expected time.

The variances of the four activities on the critical path in figure 5 were calculated as A, $\sigma^2 = 1.8$; D, $\sigma^2 = 4$; G, $\sigma^2 = 2.8$; H, $\sigma^2 = 2.8$. As stated previously, the variance for the total distribution, which is a normal probability distribution is the sum of the four individual variances, which in this case is 11.4. The standard deviation (σ) of the total distribution is therefore $\sigma = \sqrt{\sigma^2} = \sqrt{11.4} = 3.4$ days.

Knowing the required completion time, makes it possible to calculate the probability of actually completing the project before (or even after) this time. In order to find the probability of actually completing a project before its required completion time, the following formula is used:

$$Z = \frac{X - \mu}{\sigma}$$

The explanations of the elements in the formula are as follows:

X = The required completion time (LF) for the project.

μ = The earliest expected finish time for the project (mean of the normal distribution).

σ = The standard deviation of the activities on the critical path leading to project completion.

Z = The number of standard deviations between X and μ .

Referring back to figure 5, it was determined that T_E (A-D-G-H), has an expected duration of 48 days. Remember that this T_E (48 days) has a 50 percent chance that this path will be late. Now, let us evaluate the chance (probability) of completing this project in 52 days or less. Let us frame this as a question in order to answer question 2 of this paper. What is the probability that the project will be completed in 52 days or less? To find the probability, we need to find the Z using the previous formula:

$$Z = \frac{X - \mu}{\sigma}$$

Using the data we obtained from figure 5, this will be

$$Z = \frac{52-48}{3.4} = 1.18.$$

Since Z measures the number of standard deviations between μ and X on the normal probability curve, this Z value must be converted into a number that gives the proportion of the area under the normal curve that lies between μ and X . The calculation of Z

made earlier shows X to be approximately 1.18 standard deviation above the expected critical time (μ), for the project. Because the total area under the normal curve is equal to 1.0 (100 percent of all times), the probability of finishing the project before its required completion time is equal to the proportion of the area under the curve that is to the left of X . We know that the μ divides the area under the curve into two equal parts, each containing half of the area. Given this background, we know that the proportion of the area to the left of μ is equal to 0.5. Therefore, we must find the proportion of the area between μ and X and add this to 0.5 to obtain the proportion of the area to the left of X .

The previous X value indicates that there are 1.18 standard deviations (1 standard deviation = 3.4 days) between μ and X . Using a standard conversion Z table, we find that 1.18 is 0.38100. This means that for a Z value of 1.18, the proportion of the area under a normal curve is 0.38100. This number tells us that the probability of actually completing the project between μ and X , or in 48 to 52 days is 0.38100; thus there is a 38.10 percent likelihood. However, because we are interested in finding the probability of actually completing the project any time before 52 days, we must add the probability of finishing before 48 days. Therefore, the probability of finishing the project any time before 52 days is equal to the probability of finishing before 48 days plus the probability of finishing between 48 days and 52 days which is: $0.50000 + 0.38100 = 0.88100$.

Given the above computation, the probability of actually completing the project before its required completion time of 52 days is 0.88100. Probabilistically, we can say that there is an 88.100 percent likelihood (or chance).

Secondly, What is the probability of completing the above project before a required completion time of 40 days if the earliest expected finish time is still 48 days and the standard deviation still remains at 3.4 days? In this case:

$$Z = \frac{X - \mu}{\sigma}; \quad Z = \frac{40 - 48}{3.4} = -2.35.$$

Again, using a standard conversion Z table, we find that the probability of completing the project between μ and X (i.e., between 40 days and 48 days) is 0.49061. To determine the probability of completing the project before its required completion time of 40 days, we need to subtract 0.49061 from 0.50000 which is 0.00939. Therefore, there is a .939% probability of completing the project before its required completion time of 40 days.

Question 3

When is it appropriate to use a particular scheduling technique in a given project situation?

There are two scheduling techniques widely used in project management. These are the PERT/ADM and the CPM/PDM. With regard to activity estimating, Elmaghraby (1990) describes CPM and PERT as project management approaches using deterministic and probabilistic activity duration estimating methods.

The most widely used scheduling technique between the two is the CPM/PDM. CPM is considered to be deterministic in nature while PERT is considered to be probabilistic or stochastic in nature (Briggs, 2014; Gido, Clements, & Baker, 2014). In deterministic scheduling the risks are handled as static entities. The task and project duration are considered fixed values. Therefore, this type of scheduling is mostly used where the projects done are similar in nature and the project manager has an end to end visibility of the projects. Using the deterministic scheduling technique requires experience and reference to previous projects that are similar in deed to the one under consideration. Further, where CPM is used, the better visibility lets the project manager do a confident risk assessment. While developing the schedule, the project manager takes into account all those factors, which give him/her the confidence over the project plan. In probabilistic scheduling technique, on the other hand, risks are stochastic processes having probabilistic outcomes. The project duration is not a fixed value, but a value determined from the probability distribution with some confidence level associated. This type of scheduling is used where there is more uncertainty in the project. While developing the project plan, the project manager has to consider various factors, which are uncertain in themselves. Probabilistic scheduling gives a realistic view of the project plan, helping project managers predict the uncertainty and its effect on the plan. Therefore, PERT technique is probabilistic or stochastic in nature.

Furthermore, Meredith, Shafer, Mantel, and Sutton (2014) made the following observations.

- 1) PDM/CPM should be used where the control of costs associated with expediting work is an important concern. PDM networks should be used where the project requires the use of leads and lags between activities. PDM is easier to draw than ADM; it is used in most project management software applications; and it tends to be preferred where the project requires crashing and fast tracking.
- 2) ADM/PERT on the other hand, should be used where the activity times are estimated using probability distributions in order to evaluate the range of uncertainty around the expected project duration.
- 3) PERT technique should be used where developing schedules is based on a range of activity durations.

Conclusion and Recommendations

In this article, the author discussed a variety of scheduling techniques focusing on the two main ones (CPM and PERT), calculated the expected duration for each activity, determined the expected completion time of the project, calculated the probability of completing the project at various times, and analyzed the difference between PERT Method and CPM. By making forward and backward pass calculations, the author identified the critical path and its associated critical activities. It was posited that accuracy of expected durations that affect the critical path depends on the optimistic, pessimistic, and most likely duration estimates with the PERT model. The network diagrams were discussed for displaying the project schedule while, the CPM and PERT techniques were discussed for analyzing project schedule.

It was highlighted that a technique like CPM lacks the consideration of uncertainty in the scheduling process. CPM assumes that time is known with a high degree of certainty. This is primarily due to the estimation based on analogous data and experience. This assumption reduces the flexibility needed in making adjustments to align the project management approach with its context. By ignoring the existence of risk, the estimation done and the times used in scheduling the project do not align with the environment in which the project is being performed. PERT model of estimating activity duration and developing a project schedule addresses the concern regarding ignoring the effect of uncertainty in the estimating process.

In probabilistic schedule using PERT, risks are considered to be stochastic processes having probabilistic outcomes. The project duration is not a fixed value, but a value determined from the probability distribution with a range of values. This type of scheduling is used where there is more uncertainty in the project. While developing the project plan, the project manager has to consider various factors, which are uncertain in themselves. It could be seen that Probabilistic scheduling gives a realistic view of the project plan, helping project managers predict the uncertainty and its effect on the plan.

CPM on the other hand, is deterministic in nature. It uses a one-time estimate for each activity. The context that would call for the use of either CPM or PERT was discussed. Besides, different types of network diagramming techniques were developed, presented from different project times were analyzed. PERT was used to forecast or predict the probability of different project completion times.

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