Global Scientific JOURNALS
GSJ: Volume 10, Issue 4, April 2022, Online: ISSN 2320-9186
www.globalscientificjournal.com

TRIANGLE'S COMMON FORMULA OF HEIGHT AND AREA

Nrusingh Charan Mohapatra , M.Sc., M.Phil.
Rtd Reader in Mathematics , B . P . College, Boudh
At/Po/Dist – Boudh , State – Odisha , Country – India, Pin – 762014
Mobile No. – 9438037166 , E-mail Id – nrusingh1957@gmail.com

ABSTRACT:

A triangle is a polygon with three edges and three vertices. The area is the amount of space taken up by a two dimensional shape . The height is the perpendicular drawn from any vertex to any opposite side base of any triangle. There are three types of triangle based on angles and three types of triangle based on sides .Scalene Triangle, Isosceles Triangle and Equilateral Triangle are based on sides. Acute Triangle, Obtuse Triangle and Right Triangle are based on angles. There is a common height formula on any side of any triangle and also a common area formula of it .If ABC is any triangle then AD , BE and CF are the heights on the sides of BC, AC and AB respectively.

Hence Common Height = (1/ 2 base) $\sqrt{(\boldsymbol{a}+\boldsymbol{b}+\boldsymbol{c})(\boldsymbol{b}+\boldsymbol{c}-\boldsymbol{a})(\boldsymbol{c}+\boldsymbol{a}-\boldsymbol{b})(\boldsymbol{a}+\boldsymbol{b}-\boldsymbol{c})}$

Here Base is any side of the triangle on which the height is calculated

And Common Area = (¼) $\sqrt{(\boldsymbol{a}+\boldsymbol{b}+\boldsymbol{c})(\boldsymbol{b}+\boldsymbol{c}-\boldsymbol{a})(\boldsymbol{c}+\boldsymbol{a}-\boldsymbol{b})(\boldsymbol{a}+\boldsymbol{b}-\boldsymbol{c})}$

KEY WORDS:

Height, Area , Scalene triangle, Isosceles triangle, Equilateral triangle, Right triangle, Acute triangle, Obtuse triangle

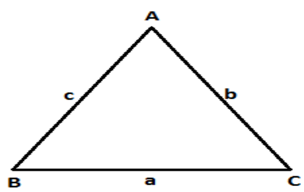
SUBJECT MATTER:

There are various types of triangle such as Acute Triangle, Obtuse Triangle, Right Triangle, Isosceles Triangle, Equilateral Triangle, and Scalene Triangle . Every triangle has three heights, which are called the altitudes. Each triangle has separate height formula but scalene triangle has no height formula at all . The area of the scalene triangle is found out by Heron's formula as follows ,

Area = $\sqrt{\boldsymbol{s}(\boldsymbol{s}-\boldsymbol{a})(\boldsymbol{s}-\boldsymbol{b})(\boldsymbol{s}-\boldsymbol{c})}$ --------(1)

Here ABC is a triangle , where AB = c , BC = a , CA = b and 2s = a + b+ c

This implies that $s = \frac{a+b+c}{2}$



The most popular area formula of a triangle is as follows

Area = ½ (base . height) -----------------(2)

Where b = base of the triangle

And h = height of the triangle

The common height formula on any side of any triangle can be found out as follows.

The height of any triangle on any side AB or BC or AC as base can be found out .

Combining the Area equations of (1) and (2) ,

It is found as follows

Area $=\sqrt{s(s-a)(s-b)(s-c)}$

$= (\frac{1}{2})$ base . height

This implies that

Area = (½) base . height

$= \sqrt{\boldsymbol{s(s-a)(s-b)(s-c)}}$

Hence **(½) base . height**

$= \sqrt{\boldsymbol{s(s-a)(s-b)(s-c)}}$ --------------(3)

Multiplying and dividing $\sqrt{16}$ on the right hand side of the equation (3) ,

It is obtained that

(½) **base . height**

$=(\sqrt{16}\ /\sqrt{16})\sqrt{\boldsymbol{s(s-a)(s-b)(s-c)}}$

$= \sqrt{16}\ \sqrt{\boldsymbol{s(s-a)(s-b)(s-c)}}\ /\sqrt{16}$

$= \sqrt{\boldsymbol{16s(s-a)(s-b)(s-c)}}\ /\sqrt{16}$

$=\sqrt{\boldsymbol{2.2.2.2\ .s(s-a)(s-b)(s-c)}}\ /\sqrt{16}$

$= \sqrt{\boldsymbol{2s.2(s-a)2(s-b)2(s-c)}}$ **/ 4**

$=\sqrt{\boldsymbol{2s(2s-2a)(2s-2b)(2s-2c)}}$ **/ 4**

This implies that

(½) base . height =(base/2) . height

$=\sqrt{\boldsymbol{2s(2s-2a)(2s-2b)(2s-2c)}}$ **/ 4**

This implies that **Height =**

(2/base) $\sqrt{\boldsymbol{2s(2s-2a)(2s-2b)(2s-2c)}}$ **/4**

Here base is any side of the triangle on which the height of it is calculated .

This implies that

Height =

(2/4base)$\sqrt{\boldsymbol{2s(2s-2a)(2s-2b)(2s-2c)}}$

=(1/2base)$\sqrt{\boldsymbol{2s(2s-2a)(2s-2b)(2s-2c)}}$

=(1/2base).

$\sqrt{\boldsymbol{(a+b+c)(a+b+c-2a)(a+b+c-2b)(a+b+c-2c)}}$

(Putting the value **2s = a + b + c**)

It is found that

Height =

(1/2base) $\sqrt{\boldsymbol{(a+b+c)(b+c-a)(c+a-b)(a+b-c)}}$

It is the height formula of any triangle on any side of it taken as a base .

Hence It is the common Height formula of SCALENE TRIANGLE , RIGHT TRIANGLE , EQUILATERAL TRIANGLE , ACUTE TRIANGLE, OBTUSE TRIANGLE AND ISOSCELES TRIANGLE

The area of any triangle can be found out by using this commom height formula as follows,

It is obtained that

Area = (½) base . height = (base /2) . height

So Area = (base/2).height = (base/2).

(1/2base). $\sqrt{\boldsymbol{(a+b+c)(b+c-a)(c+a-b)(a+b-c)}}$

= (¼) $\sqrt{\boldsymbol{(a+b+c)(b+c-a)(c+a-b)(a+b-c)}}$

{ As Height =

(1/ 2 base)$\sqrt{\boldsymbol{(a+b+c)(b+c-a)(c+a-b)(a+b-c)}}$

And base is the side of a triangle on which the height is calculated }

This implies that ,

Area =

(¼)$\sqrt{\boldsymbol{(a+b+c)(b+c-a)(c+a-b)(a+b-c)}}$

Hence,

It is the common Area formula of SCALENE TRIANGLE , RIGHT ANGLED TRIANGLE , EQUILATERAL TRIANGLE ACUTE TRIANGLE , OBTUSE TRIANGLE AND ISOSCELES TRIANGLE .

APPLICATION :

The height and area of all the triangles can be found out by using the following common height formula as well as the common area formula .

Hence Common Height =

(1/ 2 base) $\sqrt{(a+b+c)(b+c-a)(c+a-b)(a+b-c)}$

And Common Area =

(¼) $\sqrt{(a+b+c)(b+c-a)(c+a-b)(a+b-c)}$

CASE – (I)

SCALENE TRIANGLE:

EXAMPLE -

Suppose the sides of a scalene triangle are AB = c = 9 , BC = a = 10 and CA = b = 11

Let BC = a , taken as the base .

So height of the scalene triangle on the base BC is AD

So AD =

(1/ 2 base) $\sqrt{(a+b+c)(b+c-a)(c+a-b)(a+b-c)}$

$=1/2a\sqrt{(a+b+c)(b+c-a)(c+a-b)(a+b-c)}$

=(1/2.10).

$\sqrt{(10+11+9)(11+9-10)(9+10-11)(10+11-9)}$

=(1/20)$\sqrt{(30(10)(8)(12)}$**=1/20**$\sqrt{(3.5.2)(2.5)(4.2)(4.3)}$

= (1/20).120. $\sqrt{2}$ **=** $6\sqrt{2}$

So $6\sqrt{2}=AD=$ **height of the scalene triangle on the side BC as base .**

Area of the scalene triangle =

(¼) $\sqrt{(a+b+c)(b+c-a)(c+a-b)(a+b-c)}$

=(1/4).

$\sqrt{(10+11+9)(11+9-10)(9+10-11)(10+11-9)}$

= (1/4)$\sqrt{(30(10)(8)(12)}$**=(¼). 120** $\sqrt{2}$ **= 30** $\sqrt{2}$

CASE- (II)

RIGHT TRIANGLE:

EXAMPLE - Suppose the sides of a right angled triangle are AB = c = 3 , BC = a = 4 and CA = b = 5

Let AB = c , taken as the base .

So the height of the right angled triangle on the base AB is CF

So CF =

(1/ 2 base) $\sqrt{(a+b+c)(b+c-a)(c+a-b)(a+b-c)}$

=1/2c$\sqrt{(a+b+c)(b+c-a)(c+a-b)(a+b-c)}$

=1/2.3$\sqrt{(4+5+3)(5+3-4)(3+4-5)(4+5-3)}$

=1/6$\sqrt{(4+5+3)(5+3-4)(3+4-5)(4+5-3)}$

=(1/6)$\sqrt{(12)(4)(2)(6)}$ **= (1/6) . 24 = 4**

So 4 = CF = height of the right triangled triangle on the side AB as base

Area of the right angled triangle =

(¼) $\sqrt{(a+b+c)(b+c-a)(c+a-b)(a+b-c)}$

= (1/4)$\sqrt{(4+5+3)(5+3-4)(3+4-5)(4+5-3)}$

= (1/4)$\sqrt{(12)(4)(2)(6)}$ **= (1/4)**$\sqrt{(12)(4)(2)(6)}$

= (1/4).24 = 6

CASE - (III)

EQUILATERAL TRIANGLE:

EXAMPLE -

Suppose the sides of an equilateral triangle are AB = c = 4 , BC = a = 4 and CA = b = 4

Let CA= b , taken as the base ,

So the height of the right triangle on the base CA is BE

So BE =

(1/ 2 base) $\sqrt{(a+b+c)(b+c-a)(c+a-b)(a+b-c)}$

=1/2b$\sqrt{(a+b+c)(b+c-a)(c+a-b)(a+b-c)}$

=1/2.4$\sqrt{(4+4+4)(4+4-4)(4+4-4)(4+4-4)}$

=1/8 . $\sqrt{(12)(4)(4)(4)}$ **= 1/8** . $\sqrt{(4.3)(4)(4)(4)}$

=1/8 . $\sqrt{(4.3)(4)(4)(4)}$ **= (1/8) .** $4.4\sqrt{(3)}=2\sqrt{3}$

So $2\sqrt{3}=BE=$ **height of the equilateral triangle on the side CA as base**

Area of the equilateral triangle =

(¼) $\sqrt{(a+b+c)(b+c-a)(c+a-b)(a+b-c)}$

$= (¼)\sqrt{(4+4+4)(4+4-4)(4+4-4)(4+4-4)}$

$=1/4.\sqrt{(12)(4)(4)(4)}$ $=1/4.\sqrt{(4.3)(4)(4)(4)}$

$= (1/4) . 4.4\sqrt{(3)} = 4\sqrt{3}$

CASE - (IV)

ISOSCELES TRIANGLE:

EXAMPLE - Suppose the sides of a isosceles triangle are AB = c = 4 , BC = a = 4 and CA = b = 6

Let CA = b , taken as the base .

So the height of the isosceles triangle on the base CA is BE =

(1/ 2 base) $\sqrt{(a+b+c)(b+c-a)(c+a-b)(a+b-c)}$

$=1/2b\sqrt{(a+b+c)(b+c-a)(c+a-b)(a+b-c)}$

$=1/2.6\sqrt{(4+6+4)(6+4-4)(4+4-6)(4+6-4)}$

$=1/12\sqrt{(14)(6)(2)(6)} = (1/12)12\sqrt{7} = \sqrt{7}$

So $\sqrt{7} = BE =$ **height of the isosceles triangle on the side CA as base**

Area of the isosceles triangle =

(¼) $\sqrt{(a+b+c)(b+c-a)(c+a-b)(a+b-c)}$

$=(1/4)\sqrt{(4+6+4)(6+4-4)(4+4-6)(4+6-4)}$

$=(1/4)\sqrt{(14)(6)(2)(6)}$ $=(1/4) . 12\sqrt{7} = 3\sqrt{7}$

CASE - (V)

ACUTE TRIANGLE:

EXAMPLE – The formula of area and height of the ACUTE TRIANGLE is Same as the SCALENE TRIANGLE

Hence Common Height =

(1/ 2 base) $\sqrt{(a+b+c)(b+c-a)(c+a-b)(a+b-c)}$

And Common Area =

(¼) $\sqrt{(a+b+c)(b+c-a)(c+a-b)(a+b-c)}$

CASE - (VI)

OBTUSE TRIANGLE:

EXAMPLE – The formula of area and height of the OBTUSE TRIANGLE is Same as the SCALENE TRIANGLE

Hence Common Height =

(1/ 2 base) $\sqrt{(a+b+c)(b+c-a)(c+a-b)(a+b-c)}$

And Common Area =

(¼) $\sqrt{(a+b+c)(b+c-a)(c+a-b)(a+b-c)}$

CONCLUSION :

The above common height formula as well as common area formula of all the triangles are very easy as well as very simple. Hence one can memorise it easily.