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Using Information Technology in Determining Yearly Aggregate Loss Distribution in an Insurance Portfolio

Key Words: Determining Yearly Aggregate Loos Distribution through Information Technology

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Abstract

This article shows the using of information technology in preparing the claims aggregate loss table in an insurance portfolio or insurance policy and and will compare the results with both the four moments for one insurance policy introduced by HON SHIANG LAU and the for moments of all insurance portfolio introduced by THOMAS AUIPPA, hence the measurements of skewness and kurtosis.

Introduction

In general insurance work, we have to get the claim frequency distribution and claim size distribution which express the severity of claims in order to estimate the maximum probable yearly aggregate loss (MPY) of an insurance portfolio. We cant's use the traditional approach for estimating MPY because of the large portfolio size, the following equation will clear this:

$$SS = \sum_{i=0}^{n} C^{i} \quad \text{-------------------}(1)$$

Where SS denotes to the sample size, C is denoting the number of claim size distribution classes and n denotes the maximum number of claims. For example, if we have a claim frequency table consists of four rows and two columns like this: [3, 39]

Number of Claims	Number of policyholders making	Relative Frequency
0	17353	0.893931589
1	1414	0.072841541
2	620	0.031939007
3	25	0.001287863
Sum	**19412**	**1**

Also, we have the following claim size frequency distribution: [3, 53]

Mean Claim Size $	Number of policyholders making	Relative Frequency
1000	488	0.579572447
3000	115	0.136579572
5000	92	0.109263658
7000	54	0.064133017
9000	33	0.039192399
11000	19	0.022565321
13000	15	0.017814727
15000	15	0.017814727
17000	7	0.008313539

19000	4	0.004750594
SUM	**842**	**1**

The sample space of aggregate loss distribution for one insurance policy will be according to equation (1):

$$SS = \sum_{i=0}^{n} C^{i}$$

$$SS = \sum_{i=1}^{3} 10^{i} = 10^{0} + 10^{1} + 10^{2} + 10^{3} = 1 + 10 + 100 + 1000 = 1111$$

Through A computer program, we can summarize the previous aggregate loss table into 49 rows table, eventually we calculate the sample space of an insurance portfolio consists of N policies according to the following equation:

$$SSP = M^{N} \text{ --------------------(2)}$$

Where M denotes the number of aggregate loss table rows, while N is the number of polices in an Insurance Portfolio, Applying the previous equation for ten policies:

SSP = 49^{10} = 7.97923E+16

Of course, We can't build a loss aggregate table contains 7.97923E+16 rows.

Fortunately, in 1984 HON SHINAG LAU discovered the four moments of aggregate loss distribution for one insurance policy from the ordinary loss frequency moments and claim size moments.

Also in 1988, THOMAS A. AIUPPA discovered the four moments of the entire insurance portfolio from the ordinary loss frequency moments through these moments only, we can use Pearson Family Curves as a good approximation to Maximum Probable Yearly Aggregate Loss (MPY) that's because Pearson Family Curves approximation to MPY methodology depends upon the moments of an insurance portfolio.

In this research I will introduce a computer program used in preparing the annual aggregate loss table for one insurance policy and also for a number of insurance policies.

Current Study

I developed a computer used in preparing aggregate loss table for one insurance policy in an insurance portfolio and for a number of policies in order to determine an exact loss distribution and I will verify the results obtained from the program through HON SHIANG LAU and THOMAS A. AUIPPA equations of moments.

Lets' assume have the following observed numbers of policyholders making 0, 1, 2 claims which is called claim frequency table: [1, 259]

Number of claims	Relative Frequency
0	0.8
1	0.15
2	0.05

SUM	1

The four moments of the claims frequency for one policy can be calculated as follows:

$$\mu_{\text{n}} = 0.25, \quad \mu_2(n) = 0.2875, \quad \mu_3(n) = 0.31875, \quad \mu_4(n) = 0.51953125$$

Let's assume we have the following claim size frequency distribution resulting from those policyholders: [1, 236]

Claims size $	Relative Frequency
10000	0.6
20000	0.3
40000	0.1
SUM	**1**

The four moments of the claims frequency for one policy can be calculated as follows:

$$\mu_{\text{x}} = 16000, \quad \mu_2(x) = 84000000, \quad \mu_3(x) = 1.272\text{E}+12, \quad \mu_4(x) = 3.4032\text{E}+16$$

From the previous two tables we can prepare the claims aggregate distribution manually for one policy as follows: [1, 238]

Claims numbers	**Aggregate Losses**	**Probabilities and how they happen**
0	0	No claim probability = **0.8**
1	10000	Occurring of one claim with a size of 10000 = 0.15 x 0.6 = **0.09**
1	20000	Occurring of one claim with a size of 20000 = 0.15 x 0.3 = **0.045**
1	40000	Occurring of one claim with a size of 40000 = 0.15 x 0.1 = **0.015**
2	20000	Occurring of two claims, the size of the first 10000 and the size of the second is 10000 = 0.05 x 0.6 x 0.6 = **0.018**
2	30000	Occurring of 2 claims, the size of the first is 10000 and the size of the second is 20000 = 0.05 x 0.6 x 0.3 = **0.009**
2	50000	Occurring of 2 claims, the size of the first is 10000 and the size of the second is 40000 = 0.05 x 0.6 x 0.1 = **0.003**
2	40000	Occurring of 2 claims, the size of the first is 20000 and the size of the second is 20000 = 0.05 x 0.3 x 0.3 = **0.0045**
2	30000	Occurring of 2 the size of the first is 10000 and the size of the second is 20000 = 0.05 x 0.6 x 0.3 = **0.009**
2	60000	Occurring of 2 claims, with a first claims, the size of the first is20000 and the size of the second is 40000 = 0.05 x 0.3 x 0.1 = **0.0015**
2	80000	Occurring of 2 claims, the size of the first is 40000 and the size of the second is 40000 = 0.05 x 0.1 x 0.1 = **0.0005**
2	50000	Occurring of 2 claims, the size of the first is 40000 and the size of the second is 10000 = 0.05 x 0.6 x 0.1 = **0.003**
2	60000	Occurring of 2 claims, size of the first is 40000 and the size of the second

		is 20000 = 0.05 x 0.3 x 0.1 = **0.0015**
		Sum of Probabilities = 1

The sample space of the previous preliminary claims aggregate loss can be calculated according to the equation (1):

$$SS = \sum_{I=0}^{n} C^{n}$$

Where C denotes the number of claim size distribution classes and n denotes the maximum number of claims

$$SS = \sum_{I=0}^{2} 3^{i}$$

$$SS = 3^{0} + 3^{1} + 3^{2}$$

SS = 13

We will aggregate the probabilities according to each aggregate loss from the previous table to get the following yearly aggregate loss distribution per one unit.

Aggregate Losses $	Probability
0	0.80
10000	0.09
20000	0.063
30000	0.018
40000	0.0195
50000	0.006
60000	0.003
80000	0.005
Sum	**1**

Because the calculations are long and complex, I developed a computer program to do the yearly loss distribution per one unit and I got the following same result:

The Preliminary aggregate loss distribution of one policy

AC	AP
0	0.8000000119
10000	0.0900000036
20000	0.0450000018
40000	0.0150000006
20000	0.0180000011
30000	0.0090000005
50000	0.0030000003
30000	0.0090000005
40000	0.0045000003
60000	0.0015000001
50000	0.0030000003
60000	0.0015000001
80000	0.0005000000

Record: 1 of 13

The Final losses aggregate distribution for one policy

AC	SumOfAP
0	0.8000000119
10000	0.0900000036
20000	0.0630000029
30000	0.0180000011
40000	0.0195000009
50000	0.0060000005
60000	0.0030000003
80000	0.0005000000

Record: 1 of 8

The aggregate Loss distribution of one policy

AC	SumOfAP

0	0.8
10000	0.09
20000	0.063
30000	0.018
40000	0.0195
50000	0.006
60000	0.003
80000	0.0005
Sum	**1**

Where the variable AC denotes the aggregate losses and the variable SumOFAP denotes the probability

The four moments of claims aggregate distribution can be calculated as follows:

$\mu_1 = 4000, \quad \mu_2(l) = 94{,}600{,}005, \quad \mu_3(l) = 2.7828\text{E}+12, \quad \mu_4(l) = 1.18478\text{E}+17$

Hon Shiang Lau presented the following equations to calculate the four moments of aggregate loss distribution for one unit (policy): [4, 24]

$$\mu_l = \mu_x \mu_n$$

$$\mu_2(l) = \mu_x^2 \mu_2(n) + \mu_n \mu_2(x)$$

$$\mu_3(l) = \mu_x^3 \mu_3(n) + \mu_n \mu_3(x) + 3\mu_x \mu_2(x) \mu_2(n) \quad \text{--(3)}$$

$$\mu_4(l) = \mu_x^4 \mu_4(n) + \mu_n \mu_4(x) + 4\mu_x \mu_3(x)\mu_2(n) + +6\mu_x^2 \mu_2(x)[\mu_n \mu_2(n) + \mu_3(n)] + 3[\mu_2(x)]^2[\mu_n^2 - \mu_n + \mu_2(n)]$$

And now we will recall the previously calculated four moments for claims frequency and claim size to apply these equations on them:

$\mu_\text{n} = 0.25, \quad \mu_2(n) = 0.2875, \quad \mu_3(n) = 0.31875, \quad \mu_4(n) = 0.51953125$

$\mu_\text{x} = 16000, \quad \mu_2(x) = 84000000, \quad \mu_3(x) = 1.272\text{E}+12, \quad \mu_4(x) = 3.4032\text{E}+16$

$\mu_l = 4{,}000.00, \quad \mu_2(l) = 94{,}600{,}000.00, \quad \mu_3(l) = 2{,}782{,}800{,}000{,}000.00, \quad \mu_4(l) = 1.18478\text{E}+17$

As we saw, the results were identical from both LAU's equations or one policy loss aggregate distribution

It's very hard to prepare the annual loss aggregate losses table for the hole insurance portfolio for a reason due to the sample according to equation (2) as follows:

$SS_2 = M^N$

Where, M denotes to the number of claims aggregate losses distribution table rows for one policy and n denotes to the total number of policies in the insurance portfolio

For example, if we calculate the sample space for 10 polices under the last table of losses aggregate distribution for one policy it will be:

$SS_2 = 8^{10} = 1{,}073{,}741{,}824$, and for 8 policies

$SS_2 = 8^8 = 16{,}777{,}216$

We can calculate the four moments of an insurance portfolio as follows:

First, we calculate the four moments of claims frequency for the total number of policies (m) or insured units according to Thomas Auippa equations as follows: [2, 430]

$$\mu_N = m\mu_n$$
$$\mu_2(N) = m\mu_2(n)$$
$$\mu_3(N) = m\mu_3(n) \qquad \text{----------(4)}$$
$$\mu_4(N) = m(\mu_4(n) - 3\mu_2^2(n)) + 3m^2\mu_2^2(n)$$

Second, we calculate the four moments of all policies in the portfolio as follows:

$$\mu_L = \mu_x \mu_N$$
$$\mu_2(L) = \mu_x^2 \mu_2(N) + \mu_N \mu_2(x)$$
$$\mu_3(L) = \mu_x^3 \mu_3(L) + \mu_N \mu_3(x) + 3\mu_x \mu_2(x)\mu_2(N) \qquad \text{----------(5)}$$
$$\mu_4(L) = \mu_x^4 \mu_4(N) + \mu_N \mu_4(x) + 4\mu_x \mu_3(x)\mu_2(N) + 6\mu_x^2 \mu_2(x)[\mu_n \mu_2(N) + \mu_3(N)] + 3[\mu_2(x)]^2[\mu_N^2 - \mu_N + \mu_2(N)]$$

Let's apply these equations (4), (5) in our case assuming we have 8 polices or units of risk N = 8 just as an example:

$\mu_N = 2, \quad \mu_2(N) = 2.3, \quad \mu_3(N) = 2.55, \quad \mu_4(N) = 18.0425$

$\mu_x = 16000, \quad \mu_2(x) = 84000000, \quad \mu_3(x) = 1.272\text{E}+12, \quad \mu_4(x) = 3.4032\text{E}+16$

$\mu_L = 32{,}000.00, \quad \mu_2(L) = 756{,}800{,}000.00, \quad \mu_3(L) = 2.22624\text{E}+13, \quad \mu_4(L) = 2.45128\text{E}+18$

I developed another computer program to prepare the aggregate annual loss distribution for the insurance portfolio Based on the rule of determining the sample space for the process of throwing a number of dice, but this program had limited success up to 8 polices only due to the large sample space size which needs to a super computer to perform the calculation but the main idea behind the program still the same, and this was the output of my computer program:

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Sort & Filter | Records | Find | Window | Text Formatting

CV6_4.AC	CV6_4.Sum(	CV6_5.AC	CV6_5.Sum(	CV6_6.AC	Expr1013	Expr1014	CV6_6.Sum(	CV6_7.AC	CV6_7.Sum(	AC1	AP
0	0.8000000119	0	0.8000000119	0	0.8000000119	0	0.8000000119	0	0.8000000119	0	0.16777218
0	0.8000000119	0	0.8000000119	0	0.8000000119	0	0.8000000119	0	0.8000000119	10000	0.01887437
0	0.8000000119	0	0.8000000119	0	0.8000000119	0	0.8000000119	0	0.8000000119	20000	0.01321205
0	0.8000000119	0	0.8000000119	0	0.8000000119	0	0.8000000119	0	0.8000000119	30000	0.00377487
0	0.8000000119	0	0.8000000119	0	0.8000000119	0	0.8000000119	0	0.8000000119	40000	0.00408944
0	0.8000000119	0	0.8000000119	0	0.8000000119	0	0.8000000119	0	0.8000000119	50000	0.00125829
0	0.8000000119	0	0.8000000119	0	0.8000000119	0	0.8000000119	0	0.8000000119	60000	0.00062914
0	0.8000000119	0	0.8000000119	0	0.8000000119	0	0.8000000119	0	0.8000000119	80000	0.00010485
0	0.8000000119	0	0.8000000119	0	0.8000000119	0	0.8000000119	0	0.8000000119	10000	0.01887437
0	0.8000000119	0	0.8000000119	0	0.8000000119	0	0.8000000119	0	0.8000000119	20000	0.00212336
0	0.8000000119	0	0.8000000119	0	0.8000000119	0	0.8000000119	0	0.8000000119	30000	0.00148635
0	0.8000000119	0	0.8000000119	0	0.8000000119	0	0.8000000119	0	0.8000000119	40000	0.00042467
0	0.8000000119	0	0.8000000119	0	0.8000000119	0	0.8000000119	0	0.8000000119	50000	0.00046006
0	0.8000000119	0	0.8000000119	0	0.8000000119	0	0.8000000119	0	0.8000000119	60000	0.00014155
0	0.8000000119	0	0.8000000119	0	0.8000000119	0	0.8000000119	0	0.8000000119	70000	7.07789E-
0	0.8000000119	0	0.8000000119	0	0.8000000119	0	0.8000000119	0	0.8000000119	90000	1.179648E-
0	0.8000000119	0	0.8000000119	0	0.8000000119	0	0.8000000119	0	0.8000000119	20000	0.01321205
0	0.8000000119	0	0.8000000119	0	0.8000000119	0	0.8000000119	0	0.8000000119	30000	0.00148635
0	0.8000000119	0	0.8000000119	0	0.8000000119	0	0.8000000119	0	0.8000000119	40000	0.00104044
0	0.8000000119	0	0.8000000119	0	0.8000000119	0	0.8000000119	0	0.8000000119	50000	0.00029727
0	0.8000000119	0	0.8000000119	0	0.8000000119	0	0.8000000119	0	0.8000000119	60000	0.0003220
0	0.8000000119	0	0.8000000119	0	0.8000000119	0	0.8000000119	0	0.8000000119	70000	9.909045E-
0	0.8000000119	0	0.8000000119	0	0.8000000119	0	0.8000000119	0	0.8000000119	80000	4.954523E-
0	0.8000000119	0	0.8000000119	0	0.8000000119	0	0.8000000119	0	0.8000000119	100000	8.257538E-
0	0.8000000119	0	0.8000000119	0	0.8000000119	0	0.8000000119	0	0.8000000119	30000	0.00377487
0	0.8000000119	0	0.8000000119	0	0.8000000119	0	0.8000000119	0	0.8000000119	40000	0.00042467

Record: 1 of 16777216 No Filter Search

It's noted that the number of records equals to the 16,777,216 according to the equation (2) $SS_2 = M^n = 8^8$

The aggregate Loss distribution of eight policies

AC	SumOfAP
0	0.167772
10,000	0.150995
20,000	0.165151
30,000	0.126812
40,000	0.115603
50,000	0.085604
60,000	0.065531
70,000	0.043463
80,000	0.030298
90,000	0.019138
100,000	0.012303
110,000	0.00732
120,000	0.004412
130,000	0.002509
140,000	0.001435
150,000	0.000779
160,000	0.000424
170,000	0.000221
180,000	0.000116
190,000	5.81E-05
200,000	2.91E-05
210,000	1.41E-05
220,000	6.82E-06
230,000	3.19E-06

240,000	1.49E-06
250,000	6.73E-07
260,000	3.04E-07
270,000	1.33E-07
280,000	5.8E-08
290,000	2.46E-08
300,000	1.04E-08
310,000	4.25E-09
320,000	1.74E-09
330,000	6.9E-10
340,000	2.74E-10
350,000	1.05E-10
360,000	4.02E-11
370,000	1.49E-11
380,000	5.53E-12
390,000	1.98E-12
400,000	7.09E-13
410,000	2.44E-13
420,000	8.46E-14
430,000	2.79E-14
440,000	9.35E-15
450,000	2.95E-15
460,000	9.53E-16
470,000	2.87E-16
480,000	8.9E-17
490,000	2.53E-17
500,000	7.55E-18
510,000	2.01E-18
520,000	5.78E-19
530,000	1.41E-19
540,000	3.93E-20
550,000	8.62E-21
560,000	2.32E-21
570,000	4.39E-22
580,000	1.18E-22
590,000	1.69E-23
600,000	5.16E-24
610,000	3.75E-25
620,000	1.88E-25
640,000	3.91E-27
Sum	**1**

According to the well-known rule, the sum of all probabilities equals one
The symbol **AC** denotes to aggregate losses and **SumOfAP** denotes to probability
The four moments of this distribution will be:

$$\mu_L = 32{,}000.05365, \quad \mu_2(L) = 756{,}805{,}935.3, \quad \mu_3(L) = 2.22629E+13, \quad \mu_4(L) = 2.45134E+18$$

As we saw the results is very near to that of Thomas Auippa equations (5) which were:

$$\mu_L = 32{,}000 \quad \mu_2(L) = 756{,}800{,}000, \quad \mu_3(L) = 2.22624\text{E}+13, \quad \mu_4(L) = 2.45128\text{E}+18$$

The Coefficient of Skewness and Coefficient of Kurtosis according to the computer program will be:

$$\beta_1 = \mu_3^2 / \mu_2^3, \quad \beta_2 = \mu_4 / \mu_2^2 \text{----------(6)}$$

$$\beta_1 = 1.1434, \quad \beta_2 = 4.27991$$

While that of Auippa's Equations:

$$\beta_1 = 1.1434, \quad \beta_2 = 4.27987$$

It's clear that Auippa's equations treated all of insurance portfolio the same treatment notwithstanding with the heterogeneity of portfolio assumed that any policy in the portfolio has the same maximum probable aggregate loss.
My point of view in respect this subject is to not to treat the insurance portfolio as one unit that's because in the real world there are heterogeneous subpopulations, for example in case of motor insurance portfolio there are several brands ranging from popular to luxury, so we can't group them in one frequency or claim size distributions tables because each one has its maximum probable yearly aggregate losses i.e. we may have a luxury vehicle evaluated half million $ and other one evaluated 50,000 $, in this case we have to group them in two separate frequency and severity tables for pricing each kind of vehicle according to its risk factor because each of them has its own maximum probable aggregate loss.

Lau proposed the following equations to calculate the aggregate moments for grouped insurance portfolios to merge the splinted insurance portfolio into one portfolio for pricing excess of loss reinsurance treaty: [4, 28]

For w = x + y

$$\mu_w = \mu_x + \mu_y$$
$$\mu_2(w) = \mu_2(x) + \mu_2(y) \qquad \text{--------------------------------(7)}$$
$$\mu_3(w) = \mu_3(x) + \mu_3(y)$$
$$\mu_4(w) = \mu_4(x) + 6\mu_2(x)\mu_2(y) + \mu_4(y)$$

Applying the computer program on example state in page 1, we got the following results:

The Preliminary aggregate loss distribution of one policy

AC	AP
0	0.8939315677
1000	0.0422169492
3000	0.0099486662
5000	0.0079589328
7000	0.0046715480
9000	0.0028548348
11000	0.0016436927
13000	0.0012976521
15000	0.0012976521
17000	0.0006055710
19000	0.0003460406
2000	0.0107284468
4000	0.0025282202
6000	0.0020225761
8000	0.0011871642
10000	0.0007254893
12000	0.0004177059
14000	0.0003297678
16000	0.0003297678
18000	0.0001538917
20000	8.79381E-05
4000	0.0025282202
6000	0.0005957896
8000	0.0004766317
10000	0.0002797621
12000	0.0001709657
14000	9.843480E-05
16000	7.771169E-05

Record: 1 of 1111

It's noted that the program found the number of records = 1111 matches the output of equation (1).

$$SS = \sum_{i=0}^{n} C^{i}$$

$$SS = \sum_{i=1}^{3} 10^{i} = 10^{0} + 10^{1} + 10^{2} + 10^{3} = 1 + 10 + 100 + 1000 = 1111$$

It's noted that we can't prepare the previous table manually such as due to the large sample space.

The Final aggregate loss distribution of one policy

AC	SumOfAP
0	0.8939315677
1000	0.0422169492
2000	0.0107284468
3000	0.0101993885
4000	0.0050564404
5000	0.0081361852
6000	0.0046409417
7000	0.0048551205
8000	0.0033275918
9000	0.0030081804
10000	0.0023918081
11000	0.0017683924
12000	0.0016249626
13000	0.0013932144
14000	0.001261317
15000	0.0013748627
16000	0.0011330139
17000	0.0006731606
18000	0.0007290483
19000	0.0003964474
20000	0.0005022202
21000	3.822396E-05
22000	0.0002333153
23000	2.446505E-05
24000	0.0001374934
25000	1.642421E-05
26000	7.608988E-05
27000	1.055769E-05

Record: 1 of 49

The aggregate loss distribution of one policy

AC	SumOfAP
0	0.893932
1000	0.042217
2000	0.010728
3000	0.010199
4000	0.005056
5000	0.008136
6000	0.004641
7000	0.004855
8000	0.003328
9000	0.003008

10000	0.002392
11000	0.001768
12000	0.001625
13000	0.001393
14000	0.001261
15000	0.001375
16000	0.001133
17000	0.000673
18000	0.000729
19000	0.000396
20000	0.000502
21000	3.82E-05
22000	0.000233
23000	2.45E-05
24000	0.000137
25000	1.64E-05
26000	7.61E-05
27000	1.06E-05
28000	4.41E-05
29000	6.84E-06
30000	2.64E-05
31000	4.45E-06
32000	1.49E-05
33000	2.82E-06
34000	7.61E-06
35000	1.73E-06
36000	2.52E-06
37000	9.48E-07
38000	7.21E-07
39000	5.13E-07
41000	2.59E-07
43000	1.38E-07
45000	7.18E-08
47000	3.69E-08
49000	1.8E-08
51000	7.73E-09
53000	2.82E-09
55000	7.25E-10
57000	1.38E-10
Sum	**1**

The four moments of aggregate loss distribution for one policy can be calculated as follows:

$$\mu_l = 466.1617, \quad \mu_2(l) = 4{,}045{,}856.238, \quad \mu_3(l) = 50{,}262{,}934{,}045, \quad \mu_4(\mathrm{l}) = 8.00914\mathrm{E}+14$$

Comparing previously calculated four moments of claims frequency and claim size with that of Lau's equations (3), we found are very near of them as follows:

$$\mu_{\mathrm{n}} = 0.140583144, \quad \mu_2(n) = 0.192424717, \quad \mu_3(n) = 0.279192454, \quad \mu_4(n) = 0.5079749$$

$$\mu_{\mathrm{x}} = 3315.914489, \quad \mu_2(x) = 13729176.66, \quad \mu_3(x) = 98187137416, \quad \mu_4(x) = 1.21662E+15$$

$$\mu_{\mathrm{l}} = 466.16, \quad \mu_2(l) = 4{,}045{,}856.18, \quad \mu_3(l) = 50{,}262{,}932{,}691.02, \quad \mu_4(l) = 8.00914E+14$$

They are very near to each other.

$$\mu_N = 0.562332578, \quad \mu_2(N) = 0.769698866, \quad \mu_3(N) = 1.116769818, \quad \mu_4(N) = 3.364881375$$

Noting that the above moments for 4 policies were calculated from equation (4).

$$\mu_x = 3315.914489, \quad \mu_2(x) = 13729176.66, \quad \mu_3(x) = 98187137416, \quad \mu_4(x) = 1.21662\mathrm{E}+15$$

The four moments of Auippa's equations (5) for 4 policies will be:

$$\mu_L = 1{,}864.65, \quad \mu_2(L) = 16{,}183{,}424.70, \quad \mu_3(L) = 2.01052\mathrm{E}+11, \quad \mu_4(L) = 3.79294\mathrm{E}+15$$

The preliminary aggregate loss distribution according to my computer program for 4 policies is:

CV6.AC	CV6.SumOf/	CV6_1.AC	CV6_1.Sum(	CV6_2.AC	CV6_2.Sum(	CV6_3.AC	CV6_3.Sum(	AC1	AP
0	0.8939315677	0	0.8939315677	0	0.8939315677	0	0.8939315677	0	0.6385826219
1000	0.0422169492	0	0.8939315677	0	0.8939315677	0	0.8939315677	1000	0.0301578007
2000	0.0107284468	0	0.8939315677	0	0.8939315677	0	0.8939315677	2000	0.0076638972
3000	0.0101993885	0	0.8939315677	0	0.8939315677	0	0.8939315677	3000	0.0072859629
4000	0.0050564404	0	0.8939315677	0	0.8939315677	0	0.8939315677	4000	0.0036120829
5000	0.0081361852	0	0.8939315677	0	0.8939315677	0	0.8939315677	5000	0.0058121076
6000	0.0046409417	0	0.8939315677	0	0.8939315677	0	0.8939315677	6000	0.0033152702
7000	0.0048551205	0	0.8939315677	0	0.8939315677	0	0.8939315677	7000	0.0034682695
8000	0.0033275918	0	0.8939315677	0	0.8939315677	0	0.8939315677	8000	0.0023770749
9000	0.0030081804	0	0.8939315677	0	0.8939315677	0	0.8939315677	9000	0.0021489024
10000	0.0023918081	0	0.8939315677	0	0.8939315677	0	0.8939315677	10000	0.0017085951
11000	0.0017683924	0	0.8939315677	0	0.8939315677	0	0.8939315677	11000	0.0012632563
12000	0.0016249626	0	0.8939315677	0	0.8939315677	0	0.8939315677	12000	0.0011607968
13000	0.0013932144	0	0.8939315677	0	0.8939315677	0	0.8939315677	13000	0.0009952467
14000	0.001261317	0	0.8939315677	0	0.8939315677	0	0.8939315677	14000	0.0009010255
15000	0.0013748627	0	0.8939315677	0	0.8939315677	0	0.8939315677	15000	0.0009821372
16000	0.0011330139	0	0.8939315677	0	0.8939315677	0	0.8939315677	16000	0.0008093718
17000	0.0006731606	0	0.8939315677	0	0.8939315677	0	0.8939315677	17000	0.0004808742
18000	0.0007290483	0	0.8939315677	0	0.8939315677	0	0.8939315677	18000	0.0005207978
19000	0.0003964474	0	0.8939315677	0	0.8939315677	0	0.8939315677	19000	0.0002832034
20000	0.0005022202	0	0.8939315677	0	0.8939315677	0	0.8939315677	20000	0.0003587625
21000	3.822396E-05	0	0.8939315677	0	0.8939315677	0	0.8939315677	21000	2.730539E-05
22000	0.0002333153	0	0.8939315677	0	0.8939315677	0	0.8939315677	22000	0.0001666694
23000	2.446505E-05	0	0.8939315677	0	0.8939315677	0	0.8939315677	23000	1.747668E-05
24000	0.0001374934	0	0.8939315677	0	0.8939315677	0	0.8939315677	24000	9.82188E-05
25000	1.642421E-05	0	0.8939315677	0	0.8939315677	0	0.8939315677	25000	1.173268E-05

Record: 1 of 5764801 No Filter Search

It's noted that the number of records equals to the 5,764,801 according to the equation $SS_2 = M^n = 49^4$

The aggregate Loss distribution of four policies

AC1	SumOfAP
0	0.638583
1000	0.120631
2000	0.039201
3000	0.033756
4000	0.019338
5000	0.026595
6000	0.017778
7000	0.017352
8000	0.013108
9000	0.011546
10000	0.009642
11000	0.007484
12000	0.006754
13000	0.005874
14000	0.005302
15000	0.005473
16000	0.004735
17000	0.003279
18000	0.003193
19000	0.002163
20000	0.002268
21000	0.000902
22000	0.001205
23000	0.000596
24000	0.000765
25000	0.000411
26000	0.000473
27000	0.000273
28000	0.000301
29000	0.000183
30000	0.000196
31000	0.000123
32000	0.000125
33000	8.12E-05
34000	7.69E-05
35000	5.23E-05
36000	4.28E-05
37000	3.14E-05
38000	2.46E-05
39000	1.88E-05

40000	1.42E-05
41000	1.1E-05
42000	8.88E-06
43000	6.62E-06
44000	5.56E-06
45000	3.99E-06
46000	3.44E-06
47000	2.41E-06
48000	2.11E-06
49000	1.44E-06
50000	1.28E-06
51000	8.39E-07
52000	7.56E-07
53000	4.84E-07
54000	4.39E-07
55000	2.78E-07
56000	2.5E-07
57000	1.62E-07
58000	1.43E-07
59000	9.55E-08
60000	8.12E-08
61000	5.66E-08
62000	4.65E-08
63000	3.33E-08
64000	2.65E-08
65000	1.93E-08
66000	1.5E-08
67000	1.11E-08
68000	8.42E-09
69000	6.3E-09
70000	4.7E-09
71000	3.53E-09
72000	2.62E-09
73000	1.96E-09
74000	1.47E-09
75000	1.08E-09
76000	8.28E-10
77000	5.98E-10
78000	4.66E-10
79000	3.3E-10
80000	2.61E-10

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81000	1.82E-10
82000	1.45E-10
83000	9.95E-11
84000	7.93E-11
85000	5.43E-11
86000	4.3E-11
87000	2.95E-11
88000	2.31E-11
89000	1.6E-11
90000	1.23E-11
91000	8.71E-12
92000	6.53E-12
93000	4.73E-12
94000	3.46E-12
95000	2.56E-12
96000	1.83E-12
97000	1.38E-12
98000	9.62E-13
99000	7.33E-13
100000	5.05E-13
101000	3.86E-13
102000	2.64E-13
103000	2.02E-13
104000	1.38E-13
105000	1.04E-13
106000	7.2E-14
107000	5.37E-14
108000	3.75E-14
109000	2.75E-14
110000	1.95E-14
111000	1.4E-14
112000	1.01E-14
113000	7.12E-15
114000	5.2E-15
115000	3.61E-15
116000	2.65E-15
117000	1.82E-15
118000	1.33E-15
119000	9.18E-16
120000	6.64E-16
121000	4.61E-16

122000	3.28E-16
123000	2.31E-16
124000	1.6E-16
125000	1.16E-16
126000	7.8E-17
127000	5.76E-17
128000	3.78E-17
129000	2.85E-17
130000	1.82E-17
131000	1.4E-17
132000	8.75E-18
133000	6.79E-18
134000	4.19E-18
135000	3.25E-18
136000	2.01E-18
137000	1.54E-18
138000	9.62E-19
139000	7.15E-19
140000	4.61E-19
141000	3.29E-19
142000	2.21E-19
143000	1.5E-19
144000	1.05E-19
145000	6.73E-20
146000	5.01E-20
147000	3E-20
148000	2.36E-20
149000	1.33E-20
150000	1.1E-20
151000	5.89E-21
152000	5.05E-21
153000	2.6E-21
154000	2.28E-21
155000	1.15E-21
156000	1.01E-21
157000	5.13E-22
158000	4.38E-22
159000	2.31E-22
160000	1.87E-22
161000	1.04E-22
162000	7.82E-23

163000	4.74E-23
164000	3.21E-23
165000	2.15E-23
166000	1.29E-23
167000	9.69E-24
168000	5.12E-24
169000	4.31E-24
170000	2E-24
171000	1.88E-24
172000	7.72E-25
173000	8.06E-25
174000	2.99E-25
175000	3.38E-25
176000	1.17E-25
177000	1.38E-25
178000	4.72E-26
179000	5.5E-26
180000	1.95E-26
181000	2.13E-26
182000	8.28E-27
183000	7.96E-27
184000	3.56E-27
185000	2.87E-27
186000	1.53E-27
187000	9.93E-28
188000	6.56E-28
189000	3.26E-28
190000	2.76E-28
191000	1.01E-28
192000	1.14E-28
193000	2.93E-29
194000	4.57E-29
195000	7.84E-30
196000	1.79E-29
197000	1.91E-30
198000	6.84E-30
199000	4.14E-31
200000	2.53E-30
201000	7.84E-32
202000	9.02E-31
203000	1.25E-32

204000	3.09E-31
205000	1.59E-33
206000	1.01E-31
207000	1.46E-34
208000	3.15E-32
209000	7.59E-36
210000	9.19E-33
212000	2.5E-33
214000	6.27E-34
216000	1.43E-34
218000	2.9E-35
220000	5.12E-36
222000	7.6E-37
224000	8.98E-38
226000	7.63E-39
228000	3.63E-40
SUM	**1**

According to the well-known rule, the sum of all probabilities must equals one

The symbol **AC** denotes to aggregate losses and **SumOfAP** denotes to probability

The four moments of this distribution will be:

$$\mu_L = 1{,}864.647, \quad \mu_2(L) = 16{,}183{,}423.83, \quad \mu_3(L) = 2.01052\text{E}+11, \quad \mu_4(L) = 3.79294\text{E}+15$$

These results almost match that of Thomas Auippa equations which were:

$$\mu_L = 1{,}864.65 \quad \mu_2(L) = 16{,}183{,}424.70, \quad \mu_3(L) = 2.01052\text{E}+11, \quad \mu_4(L) = 3.79294\text{E}+15$$

The Coefficient of Skewness and Coefficient of Kurtosis according to the computer program will be:

$$\beta_1 = \mu_3^2 / \mu_2^3, \quad \beta_2 = \mu_4 / \mu_2^2$$

$$\beta_1 = 9.536837987, \quad \beta_2 = 14.48221549$$

Matches that of Auippa's Equations:

$$\beta_1 = 9.536836511, \quad \beta_2 = 14.48221389$$

Conclusion

This paper presented the output of a new computer program prepared by me, developed for finding the yearly aggregate loss distribution for one insurance policy and for a number of policies and compared the four moments of these aggregate losses with that of computed from Lau and Auippa equations. The result of these comparison was good, because I found the results were almost identical. The program was a complete success in setting up the aggregate loss distribution table for one insurance policy, but encounter a limited success in setting up the aggregate loss distribution for a large number of policies because the large sample space which leads to what's known The Curse of Dimensionality, but it's a step on the road. We don't have to apply Auippa's equations (4), (5) on the whole insurance portfolio because of the heterogeneity of components of the portfolio, so instead of this, we must split the portfolio according to risk factor into sub-portfolio sectors for direct insurance pricing purposes and merge the splinted sub portfolios according to Lau's equations (7) for non-proportional reinsurance treaties pricing purposes.

References

1) Ahamed, Mamdouh Hamzah, ***Mathematics of General Insurance,*** Cairo University Center for Open Education, 2005.
2) Auippa Thomas A, "Evaluation of Pearson Curves As an Approximation of the Maximum Probable Annual Aggregate Loss"***, Journal of Risk and Insurance,*** Vol.55, No.3 (Sep.,1988), pp 425-441.
3) Hossack, I.B, Polard, J.H, Zehnwirth, ***Introductory Statistics with Application in General Insurance***, CAMBRIDGE UNIVERSITY PRESS, 1983.
4) Lau, H. S., "An Effective Approach for Estimating the Aggregate Loss of an Insurance Portfolio, ***Journal of Risk and Insurance***, Vol. 51, No. 1 (1984), pp. 20-30.